

# Alternating Renewal Framework for Estimation in Spectrum Sensing Policy and Proactive Spectrum Handoff

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**Abstract**—In wireless communication, radio spectrum is a resource. Dynamic spectrum access or cognitive radio is a viable method to increase spectrum utilization. The unlicensed users access the unused portion of radio spectrum opportunistically. The unlicensed users must vacate the spectrum as soon as the licensed users appear on the channel. Since, the licensed users or Primary Users (PUs) have paid licensing fees for the spectrum, the unlicensed users or Secondary Users (SUs) must not cause interference to PU transmissions. In order that SUs can continue to operate, they must find another spectrum hole on another channel and switch to that channel. These so called, spectrum sensing and spectrum handoff, are the basic foundations of CR technology. We model the random channel as an ON-OFF process and applied renewal theory to build a framework to observe the PU activity. Then from the observations, we estimate the likelihood of PU arrival to perform spectrum handoff. We then propose a spectrum access decision making policy based on the spectrum handoff time. We then numerically evaluate our proposal and make comparisons.

**Keywords** - Cognitive Radio; Dynamic Spectrum Access; Renewal Process.

## I. INTRODUCTION

The exponential growth in the number of wireless devices since leads to dramatic increase in mobile data communication. It, in turn, increases the demand for radio spectrum. At present, the radio spectrum is allocated to licensed users by the government agencies. The static allocation means that some spectrums are under-utilized while others are congested. The uneven demand and supply ratio can be clearly observed in the most popular wireless service, the Wi-Fi or WLAN (IEEE Std. 802.11). The 2.45 gigahertz Industrial, Scientific and Medical (ISM) radio band, on which Wi-Fi devices operate, is allocated a total bandwidth of only 100 megahertz. Moreover, Federal Communication Commission (FCC) stated that more than 70% of the spectrum is under-used [1]. Underutilized empty broadcast TV channels known as “white spaces” (TVWS) across major cities were surveyed in [2].

*Dynamic Spectrum Access* (DSA) has been considered as a viable solution to solving the spectrum scarcity. *Cognitive Radio* (CR) was introduced and it became a well known DSA system [3] [4]. The basic CR architecture has two types of users; *Primary Users* (PUs) to whom the channel is licensed by the authorities and unlicensed *Secondary Users* (SUs). SUs can opportunistically access the channel without any interference to the PUs. This prioritized two-tier access introduces a new randomness to the already challenging wireless communication environment. This dynamic use of spectrum depending on PUs activity, adds new obstacles to make the network protocols adaptive to the varying available spectrum. [5]. Research for CR in TVWS for broadband usage has been conducted for more than a decade and there is an existing standard already; Regional Area Network (RAN) (IEEE 802.22 Standard) [6]. RAN focuses on broadband access network for rural areas. Their architecture is centralized with base stations controlling the air interface of fixed broadband devices. DSA for WLAN is not yet finalized and currently still under development in IEEE 802.11af task group.

Accordingly, there are two CR access methods to limit interference to the PU signal. The first one is *CR underlay access* in which SUs employ power control in PHY layer to mitigate interference to PUs [7] [17]. Another access method is *CR overlay access* [18] in which the SU network does not use power control. Since there is no power control, SUs cannot coexist in the same channel with the PUs at the same time. So, the SU network must vacate the spectrum as soon as the PU signal appears on the channel. In other words, if a PU appears on a channel, the SUs on that channel must stop transmitting and switch to another channel for further communication. This CR overlay access is compatible the IEEE 802.11 Std. because WLAN radio interface employs *Time Division Duplex* (TDD), i.e. half-duplex packet mode networks based on *Carrier Sense Multiple Access* (CSMA). It is also consistent with the asymmetry between upload and download data communication of the Internet. Based on before mentioned facts, CR overlay access is best suited for our proposal.

Whichever access method the CR employs, the SU network must have channel switching ability based on the availability of spectrum in order to recover from interruptions from the PU signals. This ability is termed as *spectrum mobility* which gives rise to a new type of handoff called *spectrum handoff*. Compared with other functionalities (spectrum sensing, spectrum management, and spectrum sharing) [5] of CR networks, spectrum mobility is less explored in the research community. However, due to the randomness of PU signal, it is difficult to achieve fast and smooth spectrum handoff. The goal of spectrum handoff for a SU network can be separated into two parts; i) to vacate the current channel with minimum delay and ii) to reestablish communication on a candidate channel which is likely to be idle also with minimum delay.

There are already a number of protocols to achieve the spectrum handoff goals which can be classified into two categories; reactive spectrum handoff protocols and proactive spectrum handoff protocols. The reactive spectrum handoff protocols use detection of the PU signal by the spectrum sensing functionality as a trigger point to start spectrum handoff procedure [8] [9]. In proactive spectrum handoff protocols, the SU network performs spectrum handoff even before the arrival of PU signal based on the estimation of previously observed channel statistics [10]. Both reactive and proactive spectrum handoff protocols observe and store channel statistics to estimate the candidate channel for the spectrum handoff process. The reactive mechanism has to detect the PU signal first before starting the handoff protocol which means it is based on collision detection and therefore some interference to PU signal is unavoidable. But the protocols provide a hard decision without any ambiguity. The proactive mechanism, on the other hand, is based on estimation of likelihood of PU signal arrival which means it can protect PU signal from any interference within an error margin. But PU signal activity statistics is needed for estimation which can take valuable time to converge to an acceptable error margin.

We, now, propose a cognitive radio (CR) framework based on the proactive spectrum handoff protocol. The proactive protocol estimates the current channel at every decision cycle and switch to candidate channel before PU signal appears. We will employ overlay CR access method to operate in IEEE 802.11 environment. The rest of this paper is organized as follows: The system model is introduced in Section II. In Section III, the details of the estimation of likelihood of channel statistics for dynamic spectrum sensing and proactive spectrum handoff framework are given. In Section IV, spectrum decisions for the proposed framework are given. Performance evaluation of our proposed protocol framework is given in Section V, followed by the conclusions in Section VI.

## II. SYSTEM MODEL

### A. Network Model

We will consider the conventional WLAN architecture for our proposal. There is an SU Access Point (SU-AP) or wireless router acting as a gateway which is connected to the backbone network. There are  $nSU$  number of SUs which the SU-AP is serving as shown in Fig. (1). The SU devices can be any wireless device with a Wi-Fi interface. Without loss of generality, we associate a PU signal with each channel licensed

to a PU as depicted in Fig. (1). The handoff procedure is performed by the SU-AP, which means the whole network will switch to another channel. This ensures that network coordination between SUs can be achieved by Distributed Coordination Function (DCF). Our goal is to keep the network coordination unchanged as much as possible. To achieve this, the SU network must stay on one channel for a beacon interval at the minimum. The default time interval for beacon interval for IEEE 802.11 is 100 Time Units (TU) where 1 TU = 1024 micro-seconds. Some control messages for CR functionalities can be piggybacked onto the beacon messages or they can be exchanged just after the beacon message. As in [6], we assumed that the channel handoff procedure initiated by the SU-AP will not require re-association and authentication.

The sequence of channel occupation by the SU network is shown in Fig. (1). One block represents one beacon interval and blocks highlighted by yellow represents that a collision between PU packets and SU packets has occurred in that beacon interval. Whenever each and every SU node wants to transmit, they will perform carrier sensing according to CSMA and they will detect the PU signal if it is present. Therefore, the collision can only happen in a scenario when the PU signal appears on the channel, which was observed as idle, during SU network operation. When that happens there will be a collision between PU packets and SU packets. For any data transmission following CSMA/CA procedure, an acknowledgement (ACK) message must be transmitted and received to verify successful data transmission. Therefore, when no ACK message was received, the SU nodes will notice there has been a collision and perform back-off procedure. So, the SU network will detect the presence of PU signal and stop the transmission. The interference to the PU signal is at most the duration of a data packet transmission [11].

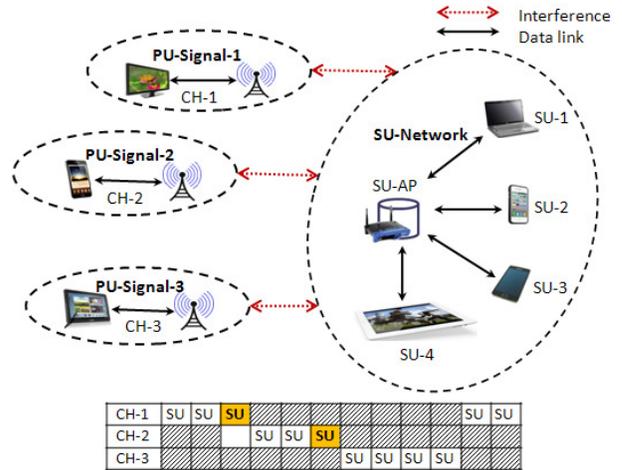


Figure 1. SU network model and channel sequence of SU

### B. Spectrum Sensing Model

In order to correctly estimate the PU signal activity on the operating channels, the SU network needs a reliable spectrum sensing method and policy. There are a number of spectrum sensing techniques available to detect the PU signal [12]. Since the SU network is a WLAN, the half-duplex nature of the SU

network gives rise to a tradeoff; spectrum sensing (exploration) vs. spectrum access (exploitation) [13]. In addition, the number of channels, on which the SU network operates, means that spectrum sensing policy should be given a higher priority than the actual PHY layer spectrum sensing techniques.

We will employ the least complex sensing technique in the PHY layer, *Energy Detection* (ED). ED is chosen because each SU already has the ability to sense the carrier frequency as CSMA/CA protocol is used for network coordination. We assumed that every SU in the network has only a single WLAN interface. The process of ED is straight forward. The received signal strength is input into ED to draw one of two hypothesis: ON (busy) or OFF (idle) by using a simple decision threshold  $\xi$  [14];

$$E(w) = \frac{1}{T} \sum_{\tau} |w(\tau)|^2 \underset{\text{idle}}{\overset{\text{busy}}{\geq}} \xi$$

$$w(\tau) = \begin{cases} u(\tau), & \text{ON (busy)} \\ v(\tau) + u(\tau), & \text{OFF (idle)} \end{cases} \quad (1)$$

where  $u(\tau)$  denotes the complex-valued independent and identically distributed (i.i.d) additive white Gaussian noise (AWGN) with zero mean and variance  $\sigma_u^2$ . The PU signal,  $v(\tau)$ , is also considered as an i.i.d complex Gaussian with zero mean and variance  $\sigma_v^2$ . The probability of false alarm and miss-detection probabilities of ED are given in [14] as;

$$\eta = 1 - \Gamma(N, \xi/\sigma_u^2)$$

$$\psi = 1 - \Gamma(N, \xi/(\sigma_v^2 + \sigma_u^2)) \quad (2)$$

where  $\Gamma(x; t) \triangleq \int_0^t (e^{-y} y^{x-1} / \Gamma(x)) dy$  is the incomplete gamma function and  $\Gamma(x) \triangleq \int_0^{\infty} e^{-y} y^{x-1} dy$  is the complete gamma function.

In conventional sensing policies for CR, quiet periods in which there is no SU transmission are scheduled to detect the PU signal. In our proposal, we assumed that one SU can sense one channel while another SU transmit in another channel. This assumption is valid since every logical channel is separated by guard bands to limit interference from neighboring channels. Since every SU in the network can detect the PU signal, we can design the spectrum sensing policy so that every channel is sensed by at least one SU per beacon period. The list of channels to be sensed in each beacon period can be broadcasted by the SU-AP after the beacon message. Let's say that we have  $nCH$  number of operating channels, and a sensing time  $t_s$  is required for ED to draw a hypothesis. Then the period or sensing time duration required to sense all operating channels is:  $T_p = t_s \times nCH$ . Here, we assume that channel switching time is already included in  $t_s$ .

We now apply another feature of WLAN into our spectrum sensing model. Network Allocation Vector (NAV) [11] is the virtual carrier sensing method for CSMA/CA. We can simply

adapt the NAV to signal that one SU is exchanging data with the SU-AP and all other SUs should perform spectrum sensing. If SU-AP broadcast Channels-to-Sense-List (CHtS) with high priority channel coming first, for example,  $[1, 3, 4, 2, \dots, nCH]$ , each idle SU can sense for a duration of  $\min(T_p, T_{NAV})$ , during every data packet exchanged in the network.

We assume that both SU-AP and SUs have a finite amount of memory as it is in the real operation environment. Our goal is to design a compact database which contains as much information as possible. The output sensing hypothesis from each SU in the network can be stored in a 100 bits bitmap for each beacon interval whereas the channel index can be stored in 600 bits for  $nCH \leq 64$ . The sensed data should be sent to SU-AP in the next beacon interval to be processed. This can be achieved via piggybacking or as separate messages. But storing channel statistics in a bitmap is not practical because of different PU signal activity on different channels. For, example, PU signal on channel-1 can appear in every 10-minute intervals whereas PU signal on channel-2 can appear once every hour. For the following hypothesis sequence of a beacon time interval  $T_p: \{...000111...1110000...000111...\}$ , we can clearly see that there are only 3 useful information bit sequences. That is when a bit changes from either 0-to-1 (idle to busy) or 1-to-0 (busy to idle). We observe these sequence changes to be PU signal arrival (*PUa*) and PU signal departure (*PUD*) time-stamps. Although each time-stamp value takes more memory than each bit, over a long period of time, storage of time-stamps is cheaper and easier to manipulate. Moreover, we can fix the number of samples ( $n$  observations) to meet error probability requirement of estimation procedure. We can clearly determine the PU signal inter-arrival times by  $PUa(n+1) - PUa(n)$  and service time by  $PUD(n) - PUa(n)$ .

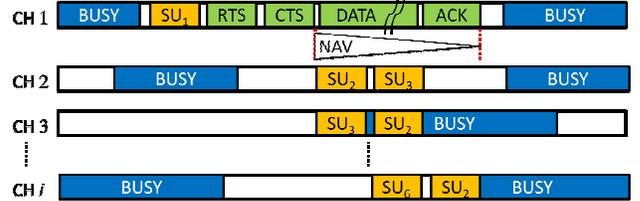


Figure 2. Independent sensing model for WLAN

### C. ON-OFF Channel Model

We use the binary ON-OFF channel to model the PU signal activity on each channel as depicted in Fig. (2). Each blue shaded rectangle represents the presence of a PU signal (i.e. ON period) and the other blank areas represent the idle intervals (i.e. OFF period). The length of the rectangle indicates the time duration the PU signal is present which is directly proportional to the PU packet length. The SU network can only utilize the idle portions of the channel.

Let  $\{X_1^i, X_2^i, X_3^i, \dots, X_{n+1}^i\}$  and  $\{B_0^i, B_1^i, B_2^i, \dots, B_n^i\}$  represent the inter-arrival times and service times of the PU signal observed by the SU network on channel- $i$  respectively. Also, let  $\{Z_0^i, Z_1^i, Z_2^i, \dots, Z_n^i\}$  be the remaining idle time of channel- $i$ . Let the random vectors  $(X_n, B_n)$ ,  $n \geq 1$  be independent and

identically distributed (i.i.d.). That is, both the sequence of random variables  $\{X_n\}$  and the sequence  $\{B_n\}$  are i.i.d. but we allow  $X_n$  and  $B_n$  to be dependent. In other words, each time the PU signal appears on channel- $i$ , everything starts over again, but when the PU signal leave channel- $i$ , we allow the length of the OFF time (idle period) to depend on the previous service time of PU signal. Let  $E[X] = E[X_n] = \mu_x$  and  $E[B] = E[B_n] = \mu_b$  denote the mean duration of the inter-arrival time and service time of PU signal, respectively. We are interested in  $P(idle)$ , the long-run proportion of time that PU signal is absent in channel- $i$ . If we let

$$X_n = B_n + Z_n, \quad n \geq 1 \quad (3)$$

then at time  $X_l$  the ON-OFF (busy-idle) cycle starts over again and repeats itself until current observation. In other words, a renewal occurs whenever a cycle is completed. Therefore, if we let  $P(busy)$  denotes the long-run busy time proportion or limiting probability, we can easily obtain as in [15].

$$P(busy) = \frac{E[B]}{E[X]} = \frac{\mu_b}{\mu_x} \quad (4)$$

$$P(idle) = 1 - P(busy) \quad (5)$$

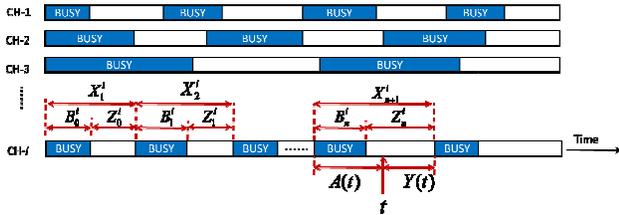


Figure 3. ON-OFF channel model based on observation by SU network

### III. CHANNEL STATISTICS ESTIMATION

Let  $F(t)$  and  $G(t)$  represent the distribution functions of  $\{X_n\}$  and  $\{B_n\}$ ,  $n \geq 1$ , respectively. If we know the type of the distribution function beforehand, we can estimate its parameters (moments) from the observed data set, and in this case, from the random vectors  $\{X_n\}$  and  $\{B_n\}$  by applying the statistical estimation methods such as *Maximum Likelihood Estimation (MLE)* [16]. We now let  $A(t)$  and  $Y(t)$  be the age and excess of the current cycle at time  $t$ , respectively. We are interested in determining the proportion of time that whether the age of a renewal cycle is less than or greater than a given time,  $c$ . For the case when the channel- $i$  is busy, i.e.  $0 < t \leq \mu_b$ , we can estimate the age of the cycle by [15];

$$\begin{aligned} A(t) &= E[\min(S, c)] \\ &= \int_0^\infty P\{\min(S, c) > s\} \\ &= \int_0^c P\{S > s\} ds \\ &= \int_0^c (1 - G(s)) ds, \quad 0 < t \leq \mu_b \end{aligned} \quad (6)$$

where  $s = \text{mod}(t, \mu_b)$ . Similarly, we can approximate the excess or residue of the cycle with;

$$\begin{aligned} Y(t) &= \int_0^c \{(1 - F(s)) - (1 - G(s))\} ds \\ &= \int_0^c \{G(s) - F(s)\} ds, \quad \mu_b < s \leq \mu_x \end{aligned} \quad (7)$$

From (6) and (7), we can determine the posterior probability that at the long run proportion of current time  $t$ , the channel- $i$  is busy conditioned on the whole renewal cycle by;

$$P(A(t) > s | s \leq \mu_x) = \begin{cases} 1 - (A(t)/\mu_x), & 0 < s \leq \mu_b \\ Y(t)/\mu_x, & \mu_b < s \leq \mu_x \end{cases} \quad (8)$$

Similarly, the posterior probability of channel- $i$  is busy at the current time  $t$ , can be determined as;

$$\begin{aligned} P(A(t) > s | s \leq \mu_b) &= P(A(t) > s | busy) \\ &= \begin{cases} 1 - (A(t)/\mu_b), & 0 < s \leq \mu_b \\ Y(t)/(\mu_x - \mu_b), & \mu_b < s \leq \mu_x \end{cases} \end{aligned} \quad (9)$$

We can similarly obtain  $P(A(t) > s | idle)$  from (6) and (7).

We now have all the values to apply *Bayesian Inference* to estimate the state of channel- $i$  in the future,  $(t+1)$ . Applying Baye's rule, we can determine the likelihoods that channel- $i$  will be busy and idle conditioned on the current age  $A(t)$  of the renewal cycle,

$$\begin{aligned} P(busy | A(t) > s) &= P(A(t) > s | busy) \times \frac{P(busy)}{P(A(t) > s)} \\ P(idle | A(t) > s) &= 1 - P(busy | A(t) > s) \end{aligned} \quad (10)$$

where  $P(A(t) > s)$  can be calculated by law of total probability as:

$$P(A(t) > s) = P(A(t) > s | busy) \cdot P(busy) + P(A(t) > s | idle) \cdot P(idle) \quad (11)$$

### IV. SPECTRUM DECISION

The purpose of the estimation is to find out how likely a channel will be idle in the future and how long it will remain idle based on previously observed channel statistics. Let us further examine the likelihood function, (10) for each channel- $i$ . The value of  $P\{busy | A(t) > s\}$  will be maximum at the start of the renewal cycle. Its value will be decreasing during the busy portion of the cycle and reach minimum value when channel- $i$  becomes idle. Then its value will increase with the length of the idle period until the end of the cycle when it will approach to one. On the other hand, the value of  $P\{idle | A(t) > s\}$  will start from zero and increases until the end of busy interval where it will reach a maximum followed by the decrease towards zero as the end of the renewal cycle approaches.

We can determine the optimal policy for *spectrum sensing*, i.e. create the Channels-to-Sense-List (CHtS). We will include channel- $i$  in the sensing list if the following condition is satisfied;

$$\begin{aligned} \text{if } P^i(busy | A(t) > s) > \epsilon_H \text{ or } P^i(busy | A(t) > s) < \epsilon_L \\ i \in CHtS[...], \end{aligned} \quad (12)$$

where  $\varepsilon_H$  is a constant and  $0 < \varepsilon_L < \varepsilon_H < 1$ . Then we can determine priority of channel sensing in CHtS by;

$$\arg \min_{i \in \text{CHtS}} f(i) := \{A^i(t) - \mu_B^i\} \quad (13)$$

Similarly, for the proactive *spectrum handoff*, the SU network will leave current channel- $i$  if the following condition is satisfied;

$$\text{if } P^i(\text{idle} | A(t) > s) < \varepsilon_L \\ i \notin \text{CHtO}[\dots] \quad (14)$$

where *CHtO* stands for *CHannel-to-Operate*. The CHtO consists of channels that are likely to be free at next beacon interval. The candidate channel- $j$  which has the least waiting time to access can be obtained as:

$$\arg \min_{j \in \text{CHtO}} f(j) := \{A^j(t) - \mu_B^j\} \quad (15)$$

## V. PERFORMANCE EVALUATION

In our framework formulation,  $F(s)$  and  $G(s)$  can be any type of distribution but we assumed that we have prior knowledge of the type of distribution beforehand. The estimation of unknown types of distribution is out of scope of this paper and it is our future work. In order to evaluate our proposal we make the following assumptions. We assume that the inter-arrival times of the PU signal on every channel is Poisson distributed with parameter  $\lambda$ . And we assume that the service times of the PU signal (i.e. the busy intervals) on every channel is normal distributed with parameters,  $\mu$  and  $\sigma$ . As we described in our ON-OFF channel,  $F(s)$  and  $G(s)$  are not independent. The distribution functions are given as:

$$F(s) = e^{-\lambda} \sum_{i=0}^{\lfloor s \rfloor} \frac{\lambda^i}{i!}, \quad s = 0, 1, 2, \dots \quad (16)$$

where  $k$  is the floor function. And,

$$G(s) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{s - \mu}{\sqrt{2\sigma^2}} \right) \right] \quad (17)$$

where  $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt$ . Simulation parameters are given in Table. I.

TABLE I. SIMULATION PARAMETERS

Parameters	Values
Number of operation channels ( $n_{CH}$ )	4
Spectrum sensing duration ( $t_s$ )	4 TU
Sensing period ( $T_p$ )	$4 \times 4 = 16$ TU
Number of observations ( $n$ )	250
Poisson inter-arrival rate ( $\lambda$ )	$1000 \times [0.25, 0.5, 0.75, 1]$ (TU)
Normal (busy) time mean ( $\mu$ )	$\lambda \times \text{rand}$ (TU)
Normal (busy) time variance ( $\sigma$ )	$0.1 \times \mu$ (TU)

We use Matlab to evaluate our proposal. First, we created 4 ON-OFF renewal channels which are observed by our proposed sensing protocol. At the initialization of the system, the observations database inside SU-AP is empty. The sensing imperfections such as delay detection are modeled into our simulation setup. We assumed that we can only stored 250

observations per channel. As described in Table I and Fig.(4), CH-1 has the shortest inter-arrival time of the 4 channels and CH-4 has the longest renewal cycle. The estimation process achieve an acceptable error margin when the target number of observations is reached, that is when current time,  $t = 2.5 \times 10^5$  TU for CH-4. Depending on the renewal cycle, channels with shorter inter-arrival times will achieve the error bound faster, for example, CH-1 can be estimated within the error margin after time,  $t = 6.25 \times 10^4$  TU.

Fig.(5) depicts the channel allocated by our proposed spectrum handoff protocol and another protocol which allocates channel solely based on likelihood threshold. As we described in Section IV, our spectrum handoff protocol is based on the duration of time (age), the SU network has to wait for an available channel if all the channels are busy. In addition, our proposed protocol chooses the candidate channel with maximum available excess time (residue).

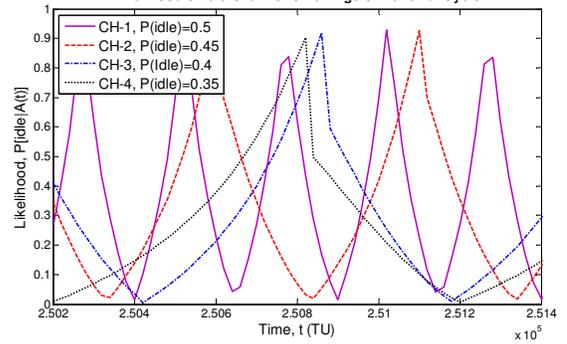


Figure 4. Likelihood of idle channel at time  $t$  for CHtO = [1, 2, 3, 4] with  $P\{\text{idle}\} = [0.5, 0.45, 0.4, 0.35]$ .

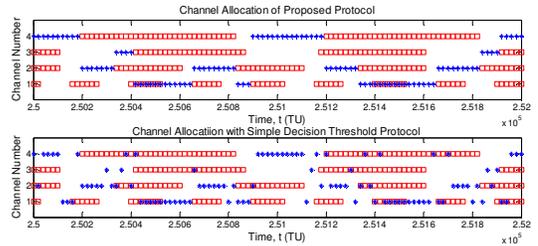


Figure 5. Comparison of channel allocations between proposed protocol and simple decision threshold protocol, (\* represents SU channel allocation, and  $\square$  represents PU signal activity) for CHtO = [1, 2, 3, 4] with  $P\{\text{idle}\} = [0.5, 0.45, 0.4, 0.35]$ .

## VI. CONCLUSION

We proposed an alternating renewal framework to estimate the PU signal activity. We formulated a new estimation protocol based on age and excess time of the renewal process for any general distribution of both inter-arrival time and service time. The framework consists of a common estimation protocol for both spectrum sensing policy and spectrum handoff. We observe that there is a major tradeoff between spectrum sensing and spectrum access policies. Our simulation results show that our proposed protocol performs better than other protocols.

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