Network Virtualization with Energy Efficiency Optimization for Wireless Heterogeneous Networks

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Abstract—In wireless network virtualization, guaranteeing service contracts with different mobile virtual network operators (MVNOs) and optimizing energy efficiency are crucial for the success of the virtualization scheme deployed by an infrastructure provider (InP). In this paper, a novel design framework is proposed for resource allocation in an OFDMA virtualized wireless network (VWN). Treating the virtual resources for a VWN as commodities, the InP wants to maximize its revenue by leasing the infrastructure and resources to the MVNOs while meeting certain contract agreements. Moreover, MVNOs want to serve their users at the best performance and pay the minimum cost to the InP. A Lyapunov based online algorithm is proposed to solve the InP's long-term optimization problem. The short-term optimization problem of the InP is considered as a combinatorial nonconvex problem. A multiple time-scale framework is proposed to solve the optimization problem of the InP, which decomposes the pricing decision, base station assignment, and resource allocation into different time-scale algorithms to achieve the design objectives. First, a distributed matching based algorithm is proposed to solve the base station assignment problem. Second, we propose a successive convex approximation approach to solve the joint subchannel assignment and energy efficiency problem. Finally, we propose a branch and bound based algorithm to optimally solve the price decision problem. Simulation results show the trade-off between energy efficiency, InP's revenue, and the isolation provisioning.

Index Terms—Network virtualization, HetNet, energy efficiency, isolation provisioning, Lyapunov optimization, matching theory

1 Introduction

Network virtualization is considered as one of the key enabling technologies to bring the forthcoming fifth generation (5G) cellular networks into realization. It is expected to achieve higher data rates, lower end-to-end latency, improve spectrum/energy efficiency, and reduce costs [2], [3], [4]. Enabling resource sharing and decoupling the infrastructure from the service it provides is the main aim of wireless network virtualization (WNV). In this case, the roles of infrastructure providers (InPs) and mobile virtual network operators (MVNOs) can be logically separated and the physical resources (e.g., spectrum, base station (BS)) owned by an InP can be transparently shared by multiple MVNOs, while each MVNO virtually owns the entire BS and radio spectrum. Network virtualization involves abstraction and sharing of resources among different parties.

With network virtualization, several benefits can be achieved through decoupling and sharing. First, the overall cost of equipment and management can be significantly reduced due to the increased hardware utilization and decoupled functionalities from infrastructure. Second, ease of migration to newer services and products as well as flexible management can be achieved. Third, WNV can have a very broad scope ranging from spectrum sharing, infrastructure virtualization to air interface virtualization.

Similar to wired network virtualization, in which physical infrastructure owned by one or more providers can be shared among multiple service providers, WNV needs physical wireless infrastructure and radio resources to be abstracted and isolated to a number of virtual resources, which can then be offered to different service providers [2], [3]. Additionally, one of the key components of the future wireless networks to improve network throughput and energy efficiency is installation of heterogeneous networks (HetNets) [6]. Therefore, to design an effective WNV, it is necessary to take HetNets into account.

A major hurdle for WNV is to achieve good resource isolation and utilization. Wireless network virtualization is a means by which an InP can slice the wireless and physical

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resources to slices. Therefore, resource allocation (RA) is considered a significant challenge for WNV, which decides how to allocate the physical resource for MVNOs to accommodate the dynamic demands of their subscribed users while satisfying the requirements of efficient RA and *isolation constraint*. The main motivation behind WNV is cost saving of network rollout, maximization of revenue for InPs and cost minimization for the MVNOs [2], [3], [4].

1.1 Related Work

Resource allocation plays an important role in achieving energy efficiency, spectrum efficiency and quality of service (QoS) provisioning in wireless networks in general. However, in WNV, isolation has to be additionally considered in RA. The problem of RA in wireless virtualization has been studied in several existing works [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19]. Wireless network virtualization in LTE is addressed in [7], [8]. In [7], the authors develop an efficient and fast centralized heuristic to allocate the radio resource block in multi-cell LTE networks based on flexible service level agreements of each service provider (SP) expressed as a minimum bandwidth allocation in each cell. The work in [8] derives a dynamic scheduling scheme while keeping track of the service contracts with the SPs and also the fairness requirement between cell-center users and cell-edge users. The work in [9] addresses the two-level hierarchical RA problem between InPs and MVNOs. A hierarchical combinatorial auction mechanism is provided, which is based on a truthful and sub-efficient RA framework. In [10], the authors propose a virtual RA scheme for OFDMA based wirless virtualization networks and show that the Pareto optimal allocation can be achieved based on the market equilibrium price theory. The problem of joint user association and power/subcarrier assignment in the multi-cell OFDMA-based virtualized wireless network is studied in [11] and [12]. In these two papers, the authors apply successive convex approximation and complementary geometric programming to convert the problem into a more tractable formulation and propose an iterative algorithm for solving the non-convex optimization problem. A Lyapunov drift-plus-penalty based algorithm for joint power and sub-carrier allocation is proposed in [13], where a minimum average required data rate of each slice and a stable-queue constraint of wireless virtualized networks are preserved. In [14], the authors virtualize the wireless network and abstract the wireless network resource as the rate region, which is computed as the set of rates that can be achieved by any spectrum allocation solutions. Another wireless resource slicing scheme called network virtualization substrate (NVS) is proposed in [15] where virtualization is achieved by modifying the MAC schedulers within the BS. Using remote traffic shaping, cell slice supports both downlink and uplink slicing. In [16], by limiting the number of mobile virtual networks embedded in the physical network, the authors show that admission control can effectively guarantee QoS experienced by users and maximize the utility of the physical network simultaneously. In [17], the authors study a virtual resource management scheme in green cellular networks with shared full duplex relaying. The problem of energy-aware virtual resource management is formulated as a three-stage Stackelberg game, and the subgame perfect equilibrium for each stage is analyzed.

However, the results in the aforementioned studies are based on the ideal backhaul assumption. In practice, the *backhaul capacity* can be limited due to the deployment costs of the backhaul links. In [18], [19] and [22], RA in virtualized wireless cellular and multi-cell OFDMA systems with backhaul capacity constraints are studied. In [18], authors investigated the virtual RA issues in small cell networks with full duplex self-backhauls and virtualization. They formulated the virtual RA problem as an optimization problem by maximizing the total utility of MVNOs. The authors in [19] formulate the RA as an optimization problem, considering not only the revenue earned by serving end users of virtual networks, but also the cost of leasing infrastructure from InPs. The authors proposed a distributed virtual RA algorithm based on the alternating direction method of multipliers (ADMM). Energy efficiency RA in multicell OFDM systems with limited backhaul capacity is studied in [22], where the trade-off between energy efficiency, network capacity, and backhaul capacity is uncovered.

1.2 Contribution

In order to overcome the "isolation" hurdle in the WNV, in this paper, we propose an efficient virtual RA framework for OFDMA based wireless virtualization networks with limited backhaul capacity, which considers the network benefits for the InP and MVNOs simultaneously. All aforementioned works either consider InP's revenue maximization or MVNO's cost minimization, but not both. Different from existing works, we consider both the revenue maximization for the InP and cost minimization for MVNOs while guaranteeing service contract agreements between the InP and MVNOs. Our network economics model has a hierarchical structure similar to the single-seller multiple-buyer hierarchical auction model in [9], where MVNO plays the role of a middleman who manages the RA for its user, while the InP is only responsible for allocating the resource to each MVNO. Different from the works in [9] and [19], where the MVNOs are also involved in making RA, in this paper, we focus on the role of the InP who is responsible for allocating resource to MVNOs' users to guarantee the service contract agreement with MVNOs in long-term evolution along with optimizing its energy efficiency for saving operation expenses. We not only consider the network model but also the *network economics*, where pricing can affect the MVNOs' demand such that RA becomes more efficient under the limitation of resource. Furthermore, we investigate the effect of backhaul link limitation on the InP's revenue and MVNOs' demand.

In this paper, we propose a dynamic resource assignment and pricing scheme which can be applied for "reseller MVNOs" and "service provider MVNOs" who almost completely rely on the InP's facilities on serving MVNOs' customers. Our proposed scheme can overcome inefficient resource utilization and unfairness between MVNOs who have heterogeneous resource demands. We focus on solving the long-term optimization problem of the InP by employing the Lyapunov optimization technique [26], which requires solving a short-term problem in each time slot. To effectively solve the short-term optimization problem of the InP, which is a combinatorial and non-convex problem, we use a locally optimal approach by decomposing the original problem into subproblems. Details of the considered problems in this paper are as follows:

(1) *MVNOs Demand Decision*: We formulate MVNOs strategy in two scenarios: independent and competition as a non-cooperative leasing game among

TABLE 1
Some Notations Used in This Paper

$\mathcal{M}$	Set of MVNOs
$\mathcal{K}$	Set of BSs
$\mathcal{U}_m$	Set of UEs of MVNO m
$\mathcal{S}_k$	Set of UEs associated with BS k
$\mathcal{U}$	Set of all UEs
$\mathcal{C}$	Set of subchannels
$a_{k,m,i}$	BS assignment variable
$y_{m,i}^{(c)}$	Subchannel assignment variable
$P_k^{(c)}$	Power variable on subchannel c of BS k
$h_{k,m,i}^{(c)}$	Channel power gain from BS k to user m_i
B_k	Backhaul constraint on BS k
η_{EE}	Energy efficiency
$r_{m,i}$	Data rate requirement of user i of MVNO m
$r_{m,i}^{\min}$	Minimum data rate requirement of user i of MVNO m
$R_{k,m,i}^{(c)}$	Achievable transmission rate of user i of MVNO m
$Q_{m,i}$	Service contract agreement queue
V	Lyapunov control parameter
β_m	Unit price applied to MVNO m
μ	Output of matching game

MVNOs. We show that there exists a unique Nash equilibrium for the non-cooperative leasing game among MVNOs.

(2) *Price Decision Problem*: In this problem, the InP determines the price charged to MVNOs based on requirements from MVNOs. This price will be calculated according to the long-term average througput of the MVNO's users and their requirements. A branch and bound based algorithm is employed to effectively solve the price decision problem.

(3) *In-slice BS Assignment*: In this problem, the physical BS is assigned to serve MVNOs' users by using the matching theory.

(4) *In-slice Subchannel Assignment and Energy Efficiency Optimization*: In this problem, the InP via the BSs simultaneously assigns the subchannels to MVNOs' users and allocates power in order to maximize the energy efficiency. The Dinkelbach method and DC (Difference of Convex functions) optimization approach are used to solve the fractional nonconvex problem.

The rest of this paper is organized as follows. Section 2 introduces WNV and system model. The problem formulation is presented in Sections 3 and 4. Section 5 presents the matching theory for the BS assignment problem. Section 6 discusses the solution approach and proposes an algorithm for joint subchannel assignment and energy efficiency optimization. Section 7 shows the pricing decision problem and proposes a branch and bound based algorithm. Numerical results are discussed in Section 8 which is followed by conclusion in Section 9. A summary of key notations is presented in Table 1.

2 System Model

2.1 Business Model

We consider the downlink of a single cell consisting single macro BS (MBS) and multiple small-cell BS (SBS) (Fig. 1). The MBS, SBSs and spectrum are owned and managed by a single infrastructure provider (InP) (i.e., monopoly market) who provides its virtual network service to a set of mobile virtual network operators $\mathcal{M}$ by individual contracts. The InP may have its own users with postpaid contract which different with MVNOs' users. We assume that MVNOs' users are served by a set $\mathcal{K}$ BSs consisting a MBS and $|\mathcal{K}|-1$ SBSs. An MVNO $m \in \mathcal{M}$ provides its service which is assumed different with the services offered by other MVNOs to a set $\mathcal{U}_m$ of subscribed users with personal contracts. Let $\mathcal{S}_k$ denote the set of users associated with BS $k \in \mathcal{K}$. Let $\mathcal{U} \triangleq \cup_{m=1}^{|\mathcal{M}|} \mathcal{U}_m = \cup_{k=1}^{|\mathcal{K}|} \mathcal{S}_k$ denote the set of all users. The InP owns a set $\mathcal{C}$ of orthogonal subchannels, each with bandwidth W. We consider a system with co-channel deployment, i.e., full frequency reuse [1] where all $|\mathcal{C}|$ subchannels are allocated to users in any BS.

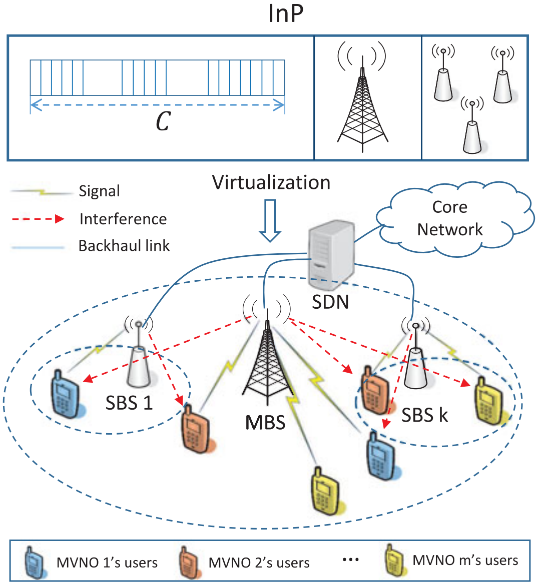


Fig. 1. System model: The InP owns the physical resource (i.e., spectrum and BSs), virtualizes these resources, and allocates to multiple MVNOs.

2.2 Wireless Network Virtualization Model

Isolation at the physical resource level can be implemented in different manners. The first is a static fixed sharing scheme in which each MVNO is preassigned a fixed subset of physical resources in different domains, and the access is restricted within this fixed subset. The second is a general dynamic sharing scheme that imposes no restriction on the resource access, while the isolation is achieved by guaranteeing certain predetermined requirements or contract service agreement (e.g., minimum share of resource or data rate) [9]. In this work, we adopt the second isolation scheme, i.e., the dynamic sharing scheme in which the InP guarantees service contract agreements with MVNOs by a long-term data rate constraint.

In-slice BS Assignment. To describe the BS assignment, let $\mathbf{A} \in \mathbb{R}^{|\mathcal{U}| \times |\mathcal{K}|}$ be the matrix for all $|\mathcal{U}|$ users over $|\mathcal{K}|$ BSs whose elements are defined as follows:

$$a_{k,m,i} = \begin{cases} 1 & \text{if BS } k \text{ is assigned for user } i \text{ of MVNO } m \\ 0 & \text{otherwise.} \end{cases}$$

We assume that at most one BS can be assigned to a user, i.e.,

$$\sum_{k \in \mathcal{K}} a_{k,m,i} \le 1, \;\; \forall i \in \mathcal{U}_m, \forall m \in \mathcal{M}. \quad (1)$$

In-slice Subchannel Assignments. To describe the subchannel assignments, let $\mathbf{Y} \in \mathbb{R}^{|\mathcal{U}| \times |\mathcal{C}|}$ be the matrix for all $|\mathcal{U}|$ users over $|\mathcal{C}|$ subchannels whose elements are defined as follows:

$$y_{m,i}^{(c)} = \begin{cases} 1 & \text{if subchannel } c \text{ is assigned for user } i \text{ of MVNO } m, \\ 0 & \text{otherwise.} \end{cases}$$

We assume that a subchannel can be assigned to at most one user in any BS

$$\sum_{m_i \in \mathcal{S}_k} y_{m,i}^{(c)} \leq 1, \ \forall k \in \mathcal{K}, \forall c \in \mathcal{C}, \tag{2}$$

where m_i denotes user i of MVNO m. To guarantee the service contract agreement, each of the MVNO's users must be assigned at least one subchannel at any time period

$$\sum_{c \in \mathcal{C}} y_{m,i}^{(c)} \geq 1, \ \forall k \in \mathcal{K}, \forall m_i \in \mathcal{S}_k. \tag{3}$$

Transmission Rate. Let $h_{k,m,i}^{(c)}$ and $\sigma_{m,i}^{(c)}$ be the channel power gain from BS k to user m_i and the noise power over subchannel c, respectively. Denote $\mathbf{P}^{(c)} = \{P_0^{(c)}, P_1^{(c)}, \ldots, P_{|\mathcal{K}|}^{(c)},\}$, $\mathbf{P}_k = \{P_k^{(1)}, P_k^{(2)}, \ldots, P_k^{(|\mathcal{C}|)},\}$, and $\mathbf{P} = \{\mathbf{P}_0, \mathbf{P}_1, \ldots, \mathbf{P}_{|\mathcal{K}|}\}$ where $P_k^{(c)}$ is the transmit power of BS k on subchannel c. Since there are cross-tier and co-tier interference due to co-channel deployment, the signal-to-interference-plus-noise ratio (SINR) achieved at user m_i due to the transmission of BS k over subchannel c can be written as

$$\Gamma_{k,m,i}^{(c)}(\mathbf{P}^{(c)}) = \frac{h_{k,m,i}^{(c)} P_k^{(c)}}{\sum_{l \in \mathcal{K}, l \neq k} h_{l,m,i}^{(c)} P_l^{(c)} + \sigma_{m,i}^{(c)}}. \tag{4}$$

Assuming that the Shannon's capacity can be achieved, the transmission rate (in bps) for user m_i on subchannel c from BS k is given by

$$R_{k,m,i}^{(c)}(\mathbf{P}^{(c)}) = W \ln(1 + \Gamma_{k,m,i}^{(c)}(\mathbf{P}^{(c)})). \tag{5}$$

Unless otherwise stated, we assume $W = 1$ without loss of generality.

Since the backhaul capacity is limited, the total scheduled transmission rate of BS k must not exceed its backhaul capacity, i.e.,

$$\sum_{m_i \in \mathcal{U}} \sum_{c \in \mathcal{C}} a_{k,m,i} y_{m,i}^{(c)} R_{k,m,i}^{(c)}(\mathbf{P}^{(c)}) \leq B_k, \ \forall k \in \mathcal{K}. \tag{6}$$

Energy Efficiency. The InP aims at optimizing the energy efficiency for serving MVNOs' subscribed users to save operational expenses, i.e., monetary payment for electric consumption. The energy efficiency metric (bits/Hz per Joule) of all BSs is defined as the ratio of the total transmission rate and the total consumed power

$$\eta_{\text{EE}} = \frac{\mathcal{R}(\mathbf{A}, \mathbf{Y}, \mathbf{P})}{\mathcal{P}(\mathbf{A}, \mathbf{Y}, \mathbf{P})} = \frac{\sum_{k \in \mathcal{K}} \sum_{m_i \in \mathcal{U}} \sum_{c \in \mathcal{C}} a_{k,m,i} y_{m,i}^{(c)} R_{k,m,i}^{(c)}(\mathbf{P}^{(c)})}{\sum_{k \in \mathcal{K}} (\sum_{m_i \in \mathcal{U}} \sum_{c \in \mathcal{C}} a_{k,m,i} y_{m,i}^{(c)} P_k^{(c)} + P_k^0)}, \tag{7}$$

where P_k^0 denotes the additional circuit power consumption of BS k during transmissions (e.g., the power dissipations in the transmit filter, mixer, frequency synthesizer, and digital-to analog converter), which is assumed to be independent of the data transmission power.

3 MVNO Model

3.1 MVNO Problem

With virtualization, each MVNO can schedule its users and determine necessary users' data rates based on its own QoS requirement. The MVNO wants to provide its users the best performance and pays the minimum to the InP by optimizing its demand according to the price offered by the InP. In this independent MVNOs model, each MVNO acts as a proxy for its users to submit data rate demand on behalf of its users to the InP. The optimization problem of MVNO m at time t is given as follows:

$$\begin{aligned} \max_{\mathbf{r}_m(t)} \ & \sum_{i \in \mathcal{U}_m} \log(1 + r_{m,i}(t)) \\ \text{s.t.} \ & r_{m,i}(t) \geq r_{m,i}^{\min}, \ \forall i \in \mathcal{U}_m, \\ & \beta_m(t) \sum_{i \in \mathcal{U}_m} r_{m,i}(t) \leq \mathfrak{B}_m^{\max}, \end{aligned} \tag{8}$$

where $\beta_m(t)$ is the unit price at time t for the bandwidth (unit/bps) applied to MVNO m, $\mathfrak{B}_m^{\max}$ is the maximum budget for MVNO m, and $r_{m,i}(t)$ is the data rate of user $i \in \mathcal{U}_m$ at time t. Since (8) is a convex optimization, we can obtain the optimal solution of the MVNO as follows.

Lemma 1. *For a given price from the InP, the optimal solution of MVNO m's problem (8) is*

$$r_{m,i}^*(t) = \left[\frac{\mathfrak{B}_m^{\max}}{|\mathcal{U}_m| \beta_m(t)}\right]_{r_{m,i}^{\min}}, \tag{9}$$

where $[\,\cdot\,]_x = \max\{\cdot, x\}$. *If the price becomes lower, then the MVNOs' users want to be served with higher transmission data rate. In contrast, if the price is higher, the required data rate is lower, but it is not less than the minimum data rate* $r_{m,i}^{\min}$.

3.2 Extension of Multiple MVNOs Model: Non-Cooperative Game Model

In case of competition among MVNOs in leasing resource and infrastructure from InP, then the interaction among MVNOs can be characterized as a non-cooperative game model where MVNOs selfishly compete with each other.

The leasing price of the InP in time slot t is assumed to be an increasing convex function of the total rate demand of all the MVNOs $r^{tot}(t) = \sum_m r_m(t)$, where $r_m(t)$ denotes the required data rate from MVNO m in time slot t. We formulate the leasing price of the InP as $\beta(t) = \gamma(t) + \theta(t) r^{tot}(t)$, where $\gamma(t)$ is the wholesale price and $\theta(t)$ is a nonnegative coefficient with the unit of $\$/\text{bps}$. The InP can determine $\gamma(t)$ and $\theta(t)$ according to the cost of serving MVNOs' users. Given the leasing price $\beta(t)$ decided by the InP, MVNOs compete with each other to maximize their own utility in a non-cooperative leasing game (NLG) defined as follows

Definition 1. *A non-cooperative leasing game* $\mathcal{G}$ *is defined as a triple:* $\mathcal{G} \triangleq \{\mathcal{M}, (r_m)_{m \in \mathcal{M}}, (U_m)_{m \in \mathcal{M}}\}$, *where* $\mathcal{M}$ *is the player set (set of MVNOs),* $(r_m)_{m \in \mathcal{M}}$ *is the strategy set, and* $(U_m)_{m \in \mathcal{M}}$ *is the payoff function.*

Without loss of generality, we assume that all users belonging to the same MVNO have the homogeneous data rate requirement $r_m^{\min}$. Then the payoff function of each MVNO m at each time slot t can be formulated as follows:

$$U_m(t) = (\alpha_m + \delta_m) r_m(t) - \beta(t) r_m(t), \tag{10}$$

where $\beta(t) = \gamma(t) + \theta(t)r^{tot}(t)$ is the leasing price from InP, α_m is the retail price set by MVNO m and δ_m is the saving cost of MVNO m (the cost saving from MVNO m since it does not need to pay to keep up a network, e.g., electric bill). The first term in the MVNO's payoff function is the revenue of MVNO from selling its services to customers, the second term is the cost for leasing resource and infrastructure from InP.

Theorem 1. *There exists a unique Nash equilibrium* $\mathbf{r}^{\rm NE} = (r_1^{\rm NE}, r_2^{\rm NE}, \ldots, r_M^{\rm NE})$ *in the proposed NLG game, where*

$$r_m^{\rm NE} = \left[\frac{1}{\theta}\left(\alpha_m + \delta_m - \gamma - \frac{1}{M+1}\left(\sum_{n\in\mathcal{M}}(\alpha_n + \delta_n) - M\gamma\right)\right)\right]_{r_m^{\min}}, \ \forall m \in \mathcal{M}. \tag{11}$$

Proof. Please see Appendix A, which can be found on the Computer Society Digital Library at http://doi.ieeecomputersociety.org/10.1109/TMC.2018.2872519/. □

Remark 1. It is observed that (i) different values of γ and θ will result in different NE; (ii) for given γ and θ, the NE is unique; (iii) all the MVNOs have different strategies at the NE depend upon their retail price α_m and saving cost δ_m; (iv) each MVNO has to know retail price and saving cost of all other MVNOs in order to update its strategy which can be done via the InP.

4 InP Model

We consider a discrete queueing model of a single InP serving set of MVNOs's users. Suppose there are virtual queues maintained for the service contract agreement (isolation constraint) for MVNOs' users, represented by $\mathbf{Q}(t) \triangleq \{Q_{m,i}(t), \forall m, i\}$, where $Q_{m,i}(t)$ denotes the virtual queue backlog for user i of MVNO m at time slot t. The data rate demand from MVNO's users $r_{m,i}(t)$ given in either (9) or (11) corresponding to independent MVNOs or competing MVNOs, respectively can be considered as the "arrival rate". Define $R_{m,i}(t) \triangleq \sum_{k\in\mathcal{K}}\sum_{c\in\mathcal{C}} a_{k,m,i} y_{m,i}^{(c)} R_{k,m,i}^{(c)}(\mathbf{P}^{(c)})(t)$ as the instantaneous rate at time t of user i of MVNO m. $R_{m,i}(t)$ can be considered as "service rate". Then, the evolution service contract agreement queue can be written as follows:

$$Q_{m,i}(t+1) = \max\{Q_{m,i}(t) - R_{m,i}(t) + r_{m,i}(t), 0\}. \tag{12}$$

By designing a control algorithm that can stabilize these contract agreement queues, we can guarantee that the required data rates of individual users can be maintained in the long run. For more details and background about this virtual queue based design, the reader can refer to [26] and references therein. Note also that the time index t in the above expression has discrete values which reflect our proposed underlying queues belong to the discrete queuing model.

The long-term optimization problem of the InP subject to the stability of the contract agreement queue (i.e., the achieved rate reaches the required minimum rate) is given as follows:

$$\begin{aligned}
&\max_{(\boldsymbol{\beta},\mathbf{A},\mathbf{Y},\mathbf{P})} \ \limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\left\{\sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{U}_m}\beta_m(t)r_{m,i}(t) + \omega\eta_{\rm EE}(t)\right\}\\
&\text{subject to:}\\
&C_1: \sum_{k\in\mathcal{K}} a_{k,m,i}(t) \le 1, \ \forall i\in\mathcal{U}_m, \forall m\in\mathcal{M},\\
&C_2: \sum_{m_i\in\mathcal{S}_k} y_{m,i}^{(c)}(t) \le 1, \ \forall k\in\mathcal{K}, \forall c\in\mathcal{C},\\
&C_3: \sum_{c\in\mathcal{C}} y_{m,i}^{(c)}(t) \ge 1, \ \forall k\in\mathcal{K}, \forall m_i\in\mathcal{S}_k,\\
&C_4: \limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\{Q_{m,i}(t)\} < \infty, \ \forall i, m,\\
&C_5: \sum_{m_i\in\mathcal{U}}\sum_{c\in\mathcal{C}} a_{k,m,i}(t) y_{m,i}^{(c)}(t) R_{k,m,i}^{(c)}(\mathbf{P}^{(c)}(t)) \le B_k, \ \forall k\in\mathcal{K},\\
&C_6: \sum_{m_i\in\mathcal{S}_k}\sum_{c\in\mathcal{C}} y_{m,i}^{(c)}(t) P_k^{(c)}(t) \le P_k^{\max}, \ \forall k\in\mathcal{K},\\
&C_7: a_{k,m,i}(t), y_{m,i}^{(c)}(t) \in \{0,1\}, P_k^{(c)}(t) \ge 0, \ \forall i, m, k, c,\\
&C_8: \beta^{\min} \le \beta_m(t) \le \beta^{\max}, \ \forall m,
\end{aligned} \tag{13}$$

where ω is the cost per unit energy efficiency. The objective of the InP here is to maximize its revenue from serving MVNOs' users, i.e., the first term, and to optimize energy efficiency, i.e, the second term. Constraint C_4 enables us to ensure that the long-term contract agreement holds, which can be considered as isolation provisioning. Constraint C_5 represents the BS's limited backhaul capacity. Hence, the InP's problem can be stated as follows: for the dynamic system defined by (12), designing a control strategy that chooses the price charged to the MVNOs, resource assignment for the MVNOs' subscribed users such that the time average revenue and energy efficiency is maximized while keeping the stability of the isolation queues.

4.1 Lyapunov-Based Online Algorithm

Since problem (13) represents a general stochastic optimization problem, we design an online algorithm based on the Lyapunov optimization techniques in [26] to convert the average based optimization problem into the single time slot optimization problem, which can be solved based on the instantaneous channel state information (CSI), queue state information (QSI) and arrival rate of each user, resulting in much less computation complexity.

Toward this end, let us consider the following Lyapunov function

$$L(\mathbf{Q}(t)) \triangleq \frac{1}{2}\sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{U}_m}\left(Q_{m,i}(t)\right)^2. \tag{14}$$

Intuitively, taking actions to push $L(\mathbf{Q}(t))$ down tends to maintain the stability of all queues, which ensures that the long-term users' rates are equal to their assigned rates (i.e., the contract agreement constraints are satisfied). Define the queuing state of the system at time t as $\mathbf{Q}(t) \triangleq \{Q_{m,i}(t), \forall m, i\}$. The one-slot conditional Lyapunov drift is defined as

$$\Delta(\mathbf{Q}(t)) \triangleq \mathbb{E}\{L(\mathbf{Q}(t+1)) - L(\mathbf{Q}(t)) | \mathbf{Q}(t)\}. \tag{15}$$

Our algorithm minimizes an upper-bound on the following *drift-plus-penalty* expression [26]

$$\Delta_V(\mathbf{Q}(t)) \triangleq \Delta(\mathbf{Q}(t)) - V\mathbb{E}\left\{\sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{U}_m}\beta_m(t)r_{m,i}(t)+\omega\eta_{\mathrm{EE}}(t)|\mathbf{Q}(t)\right\}, \tag{16}$$

where V is a non-negative weight that affects the performance bound of the algorithm. From (12) and (14), we have

Lemma 2. *For any feasible control action under constraints $C_1, C_2, C_3, C_5, C_6, C_7$, and C_8 that can be implemented at time slot t, we have the following inequality:*

$$\Delta_V(\mathbf{Q}(t)) \leq D + \sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{U}_m} Q_{m,i}(t)\big(r_{m,i}(t)-R_{m,i}(t)\big) - V\mathbb{E}\left\{\sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{U}_m}\beta_m(t)r_{m,i}(t)+\omega\eta_{\mathrm{EE}}(t)|\mathbf{Q}(t)\right\}, \tag{17}$$

where D is a constant given as

$$D \triangleq \frac{1}{2}\sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{U}_m}\left(r_{m,i}^{\max 2}+R_{m,i}^{\max 2}\right), \tag{18}$$

here $R_{m,i}^{\max}$ and $r_{m,i}^{\max}$ are finite because of the limited transmission power in (5).

Proof. Please see Appendix B, available in the online supplemental material □

Algorithm 1. Online Algorithm to Solve Problem (13)

Initialization: $V > 0, Q_{m,i}(0) \leftarrow 0, \forall m, i$, and $t \leftarrow 0$;
loop
  Collect the current QSI $Q_{m,i}(t)$;
  Collect the demand request from MVNOs' users $r_{m,i}(t)$;
  Solve the following optimization problem:

$$\max_{(\boldsymbol{\beta},\mathbf{A},\mathbf{Y},\mathbf{P})} \sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{U}_m}[V\beta_m(t)r_{m,i}(t) - (r_{m,i}(t)-R_{m,i}(t))Q_{m,i}(t)] + V\omega\eta_{\mathrm{EE}}(t)$$

subject to:

$$\begin{aligned}
&C_1: \sum_{k\in\mathcal{K}} a_{k,m,i}(t) \leq 1,\ \forall i\in\mathcal{U}_m, \forall m\in\mathcal{M},\\
&C_2: \sum_{m_i\in\mathcal{S}_k} y_{m,i}^{(c)}(t) \leq 1,\ \forall k\in\mathcal{K}, \forall c\in\mathcal{C},\\
&C_3: \sum_{c\in\mathcal{C}} y_{m,i}^{(c)}(t) \geq 1,\ \forall k\in\mathcal{K}, \forall m_i\in\mathcal{S}_k,\\
&C_5: \sum_{m_i\in\mathcal{U}}\sum_{c\in\mathcal{C}} a_{k,m,i}(t)y_{m,i}^{(c)}(t)R_{k,m,i}^{(c)}(\mathbf{P}^{(c)}(t)) \leq B_k,\ \forall k,\\
&C_6: \sum_{m_i\in\mathcal{S}_k}\sum_{c\in\mathcal{C}} y_{m,i}^{(c)}(t)P_k^{(c)}(t) \leq P_k^{\max},\ \forall k\in\mathcal{K},\\
&C_7: a_{k,m,i}(t), y_{m,i}^{(c)}(t)\in\{0,1\}, P_k^{(c)}(t)\geq 0,\ \forall i,m,k,c,\\
&C_8: \beta^{\min} \leq \beta_m(t) \leq \beta^{\max},\ \forall m;
\end{aligned} \tag{19}$$

  Update queues states $Q_{m,i}(t+1)$;
  $t \leftarrow t+1$;

We present an online algorithm to solve problem (13), which minimizes the upper-bound of the drift-plus-penalty given in Lemma 2, as summarized in Algorithm 1. The online algorithm is based on the drift-plus-penalty minimization algorithm, which is designed to operate as follows: At every slot t, all queues $Q_{m,i}(t)$ are observed. Then, the InP solves problem (19) to maximize $\sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{U}_m}[V\beta_m(t) r_{m,i}(t) - (r_{m,i}(t)-R_{m,i}(t))Q_{m,i}(t)] + V\omega\eta_{\mathrm{EE}}(t)$ or equivalently to minimize the right-hand-side of (17). Hereafter we assume $\omega = 1$ for simplicity.

4.2 Performance Bounds for the Online Algorithm

The optimization problem (19) is a nonlinear nonconvex combinatorial problem, which is difficult to solve directly. To present a solution of (19), the performance of control actions to obtain a local optima that is within an additive constant of the supremum is analyzed. Therefore, in the following, the definition of *C-additive approximation* [26] is first introduced, the basic for which the locally optimal solution of problem (19) is analyzed.

Definition 2. *For a given constant $C \geq 0$, a* C-additive approximation *of the drift-plus-penalty minimization algorithm is the choice of an action in each time slot that yields a conditional expected value on the right hand side of the drift-plus-penalty (given $\mathbf{Q}(t)$) at time slot t that is within a constant C from the supremum over all possible control actions.*

Based on Definition 1, a locally optimal algorithm can be designed and will be introduced in the following sections.

Define $f(t) \triangleq \sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{U}_m}\beta_m(t)r_{m,i}(t)+\eta_{\mathrm{EE}}(t)$, $f^{\max} \triangleq \max f(t)$, and $f^* \triangleq f(\boldsymbol{\beta}^*, \mathbf{A}^*, \mathbf{Y}^*, \mathbf{P}^*)$ where $(\boldsymbol{\beta}^*, \mathbf{A}^*, \mathbf{Y}^*, \mathbf{P}^*)$ is a theoretical optimal solution of (19) [26]. From the Lyapunov optimization approach, we state the following result.

Theorem 2. *If there are positive constants D, V, C, and ϵ such that for all time slots t and all possible values of $\mathbf{Q}(t)$, the Lyapunov drift satisfies*

$$\Delta(\mathbf{Q}(t)) - V\mathbb{E}\{f(t)|\mathbf{Q}(t)\} \leq D + C - \epsilon\sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{U}_m}Q_{m,i}(t) - Vf^*, \tag{20}$$

then time average utility and virtual queue length satisfy

$$\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{U}_m}\mathbb{E}\{Q_{m,i}(t)\} \leq \frac{D+C+V(f^{\max}-f^*)}{\epsilon}, \tag{21}$$

$$\liminf_{t\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\{f(t)\} \geq f^* - \frac{D+C}{V}, \tag{22}$$

where the values of C and ϵ depend on the performance gap between the optimal solution and the sub-optimal solution obtained by the proposed algorithmic framework.

Proof. Please see Appendix C, available in the online supplemental material. □

Remark 2.

(1) It is extremely difficult, if not impossible, to obtain the globally optimal solution to (19) with a polynomial-time algorithm. Therefore, we will focus on developing low-complexity algorithms that produce locally optimal solutions to (19) rather than global optimal solution. In general, it is difficult to quantify the gap C that the algorithms can achieve.

(2) Theorem 2 shows that the proposed online algorithm achieves an $[\mathcal{O}(1/V), \mathcal{O}(V)]$ tradeoff between

the average revenue, average energy efficiency and virtual queue backlogs [26]. With an increase of the control parameter V, the achieved performance becomes better at the cost of incurring the larger queuing delay. Therefore, it is important to choose a proper V to obtain the required performance and isolation provisioning in realistic WNV.

In the following sections, we introduce the locally optimal approach to solve (19) effectively.

4.3 Decomposition Approach for Solving Problem (19)

First, each MVNO will determine its data rate requirement as in (9) or (11) based on the current price $\beta(t)$ and send it to the InP. The InP then determines the price charged to MVNOs using a pricing decision algorithm, which will be presented in Section 7. The BS assignment and resource assignment are then implemented in different lower timescale algorithms based on the fixed price given in the pricing decision algorithm. After BS assignment is performed for all MVNOs' subscribed users (BS assignment algorithm will be presented in Section 5), the joint subchanel assignment and energy efficiency optimization will proceed distributively at each BS to assign resources (i.e., subchannels) to its associated users to achieve the design objectives, i.e., QoS guarantees for the MVNOs' users and energy-efficiency for the InP (the solution for this design is obtained by an algorithm presented in Section 6). This design allows analytical tractability to tackle problem in (19).

4.4 Stackelberg Game and Stackelberg Equilibrium

The interactions between the InP and MVNOs can be characterized as a two-stage Stackelberg game. The InP offers the price to maximize its revenue in the first stage and then the MVNOs make their decisions in setting the required data rates for their users according to the InP's price in the second stage. In case of independent MVNOs, i.e., there is no interaction between the MVNOs, the interaction takes place only on a one-by-one basis between the InP and each individual MVNO.

Let $U_{\text{InP}}(\boldsymbol{\beta}, \mathbf{A}, \mathbf{Y}, \mathbf{P}, \mathbf{r})$ denote the utility of the InP, and $U_m(\mathbf{r}_m, \beta_m)$ denote the utility of MVNO m. Denoting a solution for the InP's strategy by $(\boldsymbol{\beta}^*, \mathbf{A}^*, \mathbf{Y}^*, \mathbf{P}^*)$ and a solution for the MVNO's strategy by $(\mathbf{r}^*)$, we have the following definition:

Definition 3. $(\boldsymbol{\beta}^*, \mathbf{A}^*, \mathbf{Y}^*, \mathbf{P}^*, \mathbf{r}^*)$ *is a Stackelberg equilibrium for a Stackelberg game if it satisfies the following conditions for any values of* $(\boldsymbol{\beta}, \mathbf{A}, \mathbf{Y}, \mathbf{P}, \mathbf{r})$

$$U_{\text{InP}}(\boldsymbol{\beta}^*, \mathbf{A}^*, \mathbf{Y}^*, \mathbf{P}^*, \mathbf{r}^*) \geq U_{\text{InP}}(\boldsymbol{\beta}, \mathbf{A}, \mathbf{Y}, \mathbf{P}, \mathbf{r}^*),\ \forall \boldsymbol{\beta}, \mathbf{A}, \mathbf{Y}, \mathbf{P},$$

$$U_m(\mathbf{r}_m^*, \beta_m^*) \geq U_m(\mathbf{r}_m, \beta_m^*),\ \forall r_{m,i}.$$

For the proposed game in this paper, the Stackelberg equilibrium (SE) can be obtained as follows: For given price β, the MVNOs solve their problems first (independently or in a non-cooperative game). Then, with the required data rates of the MVNOs $(\mathbf{r}^*)$ or $(\mathbf{r}^{\text{NE}})$, the InP solves its problem for the optimal strategy $(\boldsymbol{\beta}^*, \mathbf{A}^*, \mathbf{Y}^*, \mathbf{P}^*)$. Since the problem of MVNOs is convex, the second inequality in Definition 1 is satisfied. The problem of InP in the short-time scale is nonconvex combinatorial; therefore, it is extremely difficult, if not impossible, to obtain the globally optimal solution to the InP's strategy. Since we can only obtain the locally optimal solution for the InP's strategy, it is possible to obtain more than one local SE for the proposed game.

5 In-slice Base Station Assignment

We assume that the total transmit power of BS k is divided equally among the selected subchannels, then, a UE m_i is assumed to be indifferent towards all the channels provided by that BS. This allows us to ignore the subchannel assignment variable in BS assignment problem. The BS assignment problem can be formulated as follows:

$$\begin{aligned}\max_{(\mathbf{A})}\ \ & U_{\text{InP}} = V\frac{\sum_{k\in\mathcal{K}}\sum_{m_i\in\mathcal{U}} a_{k,m,i}R_{k,m,i}(\mathbf{P})}{\sum_{k\in\mathcal{K}}(\sum_{m_i\in\mathcal{U}} a_{k,m,i}P_k + P_k^0)} \\ & \qquad + \sum_{m_i\in\mathcal{S}_k}\sum_{k\in\mathcal{K}} Q_{m,i}a_{k,m,i}R_{k,m,i}(\mathbf{P}) \\ \text{s.t.}\ \ & C_1, \\ & C_5' : \sum_{m_i\in\mathcal{U}} a_{k,m,i}R_{k,m,i}(\mathbf{P}) \leq B_k,\ \forall k\in\mathcal{K}, \\ & C_7' : a_{k,m,i}\in\{0,1\},\ \forall i,m,k,\end{aligned} \tag{23}$$

where $P_k = P_k^{\max}/|\mathcal{C}|$. The optimization problem in (23) is still an combinatorial problem; however, since (23) contains only the BS assignment binary variables, it can be modeled as a matching problem [27]. Thus, to optimally solve the optimization problem (23), we develop a distributed matching game algorithm[1] [27], [29] in the following section.

5.1 In-Slice BS Assignment as a Matching Game

Problem (23) can be modeled as a two-sided matching game where there are two disjoint sets of agents, the set of MVNO users, $\mathcal{U}$, and the set of BSs, $\mathcal{K}$. In the proposed game, each user i of MVNO m can be assigned to a single BS k. However, BS k can support a certain number of users depending upon its backhaul capacity B_k. Our design corresponds to a *one-to-many matching* given by the tuple $(\mathcal{U}, \mathcal{K}, B_k, \succ_{\mathcal{U}}, \succ_{\mathcal{K}})$. Here, $\succ_{\mathcal{U}} \triangleq \{\succ_{m_i}\}_{m_i\in\mathcal{U}_m}$ and $\succ_{\mathcal{K}} \triangleq \{\succ_k\}_{k\in\mathcal{K}}$ represent the set of the preference relations of the MVNOs' users and BSs, respectively.

Definition 4. *A* matching μ *is defined by a function from the set $\mathcal{U}\cup\mathcal{K}$ into the set of elements of $\mathcal{U}\cup\mathcal{K}$ such that:*

(1) $|\mu(m_i)| \leq 1$ *and* $\mu(m_i)\in\mathcal{K}$,
(2) $|\mu(k)| \leq \bar{\mathcal{U}}_k$ *and* $\mu(k)\in 2^{|\mathcal{U}|}\cup\phi$,
(3) $\mu(m_i) = k$ *if and only if* m_i *is in* $\mu(k)$,

where $\bar{\mathcal{U}}_k$ denotes the dynamic quota of a BS k (i.e., number of associated users to BS k) such that constraint C_5' is satisfied, and $|\mu(.)|$ denotes the cardinality of the matching outcome $\mu(.)$. The first two conditions here represent constraints C_1' and C_5' of problem (23), respectively.

5.2 Preference Function of Agents

In order to build the preference profile of MVNO user m_i, each MVNO user m_i calculates the achievable data rate for each BS k and then ranks them in descending order. The preference profile $\mathcal{P}_{m_i}$ is represented by a vector of the utility of each MVNO user m_i, as follows:

1. Employing distributed algorithms for BS assignment enables to realize a scalable and low computation solution.

$$U_{m_i}(k) = \left[R_{k,m,i}(\mathbf{P})\right]_{k\in\mathcal{K}}. \tag{24}$$

Here, each MVNO user m_i aims to be associated to a BS k such that it achieves its maximum utility $U_{m_i}(\mathcal{K})$. We use $k \succ_{m_i} k'$ to indicate that the MVNO user m_i prefers BS k to BS k', i.e., $U_{m_i}(k) > U_{m_i}(k')$.

For distributed implementation, we decouple energy efficiency optimization for all BSs into that for each BS and define the utility function of a BS k as follows:

$$U_k(m_i) = \left[V\frac{R_{k,m,i}(\mathbf{P})}{P_k} + Q_{m,i}R_{k,m,i}(\mathbf{P})\right]_{m_i\in\mathcal{U}}. \tag{25}$$

The preference profile $\mathcal{P}_k$ of BS k is defined over all MVNO users m_i which maximizes its energy efficiency while maintaining the backhaul capacity constraint. The utility function (25) reflects the energy efficiency of BS k for each user and ranks the users in the decreasing order in its preference profile $\mathcal{P}_k$. Note that, in the formulated game, the quota of BS k (i.e., $\bar{\mathcal{U}}_k$) is not static as in traditional one-to-many matching games [30]. Our game involves a dynamic quota $\bar{\mathcal{U}}_k$ as a BS k allows a number of users (with heterogeneous rates, i.e., $R_{k,m,i}$ for user m_i) as long as the backhaul constraint on that BS k is not violated, i.e., $C_5^{'}$. We denote the achievable rate of each user m_i for a BS k by $\bar{\mathcal{U}}_k^{m_i} = R_{k,m,i}$.

For the formulated two-sided matching game in Definition 2, our goal is to seek a *stable matching*, which is a key solution concept [28]. In order to have a stable matching, we should not have any blocking pair. However, dynamic quota of BSs introduces new challenges that prevent the use of standard deferred-acceptance algorithm. Therefore, we formally define the blocking pair for the formulated game as follows:

Definition 5. *A matching μ is* stable *if there exists no blocking pair (m_i, k), where $m_i \in \mathcal{U}, k \in \mathcal{K}$, such that $\bar{\mathcal{U}}_k^{res} \geq \bar{\mathcal{U}}_k^{m_i}$, $m_i \succ_k \emptyset$, and $k \succ_{m_i} \mu(m_i)$, where $\mu(m_i)$ represents the current matched partners of m_i.*

Here, $\bar{\mathcal{U}}_k^{res} = \bar{\mathcal{U}}_k - \sum_{m_i\in\mu(k)} \bar{\mathcal{U}}_k^{m_i}$ represents the residual of the quota (remaining quota) on BS k. *The quota of BS $k \in \mathcal{K}$ is filled when $\bar{\mathcal{U}}_k^{res} < \bar{\mathcal{U}}_k^{m_i}$ for a user $m_i \in \mathcal{U}$.* Definition 3 is based on the following intuition. Whenever a BS k has enough quota $\bar{\mathcal{U}}_k^{res}$ to admit a user m_i (i.e., $\bar{\mathcal{U}}_k^{res} \geq \bar{\mathcal{U}}_k^{m_i}$) and the user m_i is willing to accept k over its current matching $\mu(m_i)$ (i.e., $k \succ_{m_i} \mu(m_i)$), then k and m_i can deviate from their assigned matching to form a blocking pair. A matching is stable only if there exist no blocking pairs.

5.3 Distributed Base Station Assignment Algorithm

In order to solve this game, we propose a distributed BS assignment algorithm by modifying the deferred acceptance algorithm in [28] to produce a stable matching in Algorithm 2. After initialization of the algorithm, both sides build their respective preference profiles, i.e., $\mathcal{P}_k$ for BS k and $\mathcal{P}_{m_i}$ for user m_i. At each iteration t, each MVNO user m_i receives proposals from BS k that rank m_i as the highest in $\mathcal{P}_k[t]$ and has enough quota to admit user m_i (lines 5-8). Note that the BS k has a predefined quota of $\bar{\mathcal{U}}_k$. Each BS k continues making proposals as long as either there exist unassigned MVNO users m_i or the residual quota $\bar{\mathcal{U}}_k$ still has sufficient capacity to accommodate at least one more MVNO user m_i, i.e., $\bar{\mathcal{U}}_k^{res} \geq \bar{\mathcal{U}}_k^{m_i}$ (line 8). On receiving the proposal from BS k, each m_i: (i) temporarily accepts the proposal only if k is preferred over its current matching partner, i.e., $k \succ_{m_i} \mu[t](m_i)$ (line 9), and updates its current matching $\mu[t](m_i)$ by removing the previously accepted BS k' and accepting the new proposal by BS k. Moreover, the residual quota is also updated by both the rejected BS k' and accepted BS k (lines 10-13). Note that all the BSs k' which are ranked lower than the current matched BS k form a set represented by $R_{m_i}[t]$ (line 14); (ii) otherwise, it rejects the proposal of BS k and finds all lower ranked BSs compared to the current matching partner $\mu[t](m_i)$ and places them in the set $R'_{m_i}[t]$ (lines 15-17). Each least preferred BS $k \in \mathcal{L}_{m_i}[t]$ is then rejected, and finally, removed from the preference list $\mathcal{P}_{m_i}[t]$, and similarly these BSs also remove m_i from their respective preference list $\mathcal{P}_l[t]$ (lines 18-20). This process is repeated until the matching process converges i.e., $\mu[t] = \mu[t-1]$ (line 21). Note that by the process of BS proposing, it is guaranteed that no BS k will be matched to a user m_i, that ranks lower than the current matched user set, i.e., $\mu(k)$. In the last phase of the BS assignment process, the output $\mu[t]$ can be transformed to a feasible BS assignment vector $\mathbf{A}$ of problem (23) (line 22), i.e, $\mu \mapsto \mathbf{A}$.

Property 1. *Algorithm 2 converges to a stable assignment.*

Proof. Please see the Appendix D, available in the online supplemental material □

The optimality property[2] of the stable matching approach can be observed using the definition of weak Pareto optimality [32]. Let $u(\mu) \approxeq U_{\text{InP}}$ denote the utility obtained by matching μ. A matching μ is weak Pareto optimal if there is no other matching μ' that can achieve a better utility, i.e., $u(\mu') \geq u(\mu)$.

Algorithm 2. Distributed Base Station Assignment Algorithm

1: ***Phase 1: Initilization***:
2: **input**: $\mathcal{P}_k, \mathcal{P}_{m_i}, \forall m_i, k$;
3: **initialize**: $t = 0$, $\mu[0] \triangleq \{\mu[0](k), \mu[0](m_i)\}_{k\in\mathcal{K}, m_i\in\mathcal{U}_m} = \emptyset$, $\mathcal{L}_{m_i}[0] = \emptyset, \bar{\mathcal{U}}_k^{res}[0] = \bar{\mathcal{U}}_k$, $\mathcal{P}_k[0] = \mathcal{P}_k$, $\mathcal{P}_{m_i}[0] = \mathcal{P}_{m_i}, \forall k, m_i$;
4: ***Phase 2: Matching***:
5: **repeat**
6: $t \leftarrow t + 1$;
7: **for** $k \in \mathcal{K}$, propose m_i according to $\mathcal{P}_k[t]$ **do**
8: **while** $k \notin \mu[t](m_i)$ and $\bar{\mathcal{U}}_k^{res}[t] \geq \bar{\mathcal{U}}_k^{m_i}[t]$ **do**
9: **if** $k \succ_{m_i} \mu[t](m_i)$ **then**
10: $\mu[t](m_i) \leftarrow \mu[t](m_i) \setminus k'$;
11: $\bar{\mathcal{U}}_{k'}^{res}[t] \leftarrow \bar{\mathcal{U}}_{k'}^{res}[t] + \bar{\mathcal{U}}_{k'}^{m_i}[t]$;
12: $\mu[t](m_i) \leftarrow k$;
13: $\bar{\mathcal{U}}_k^{res}[t] \leftarrow \bar{\mathcal{U}}_k^{res}[t] - \bar{\mathcal{U}}_k^{m_i}[t]$;
14: $R_{m_i}[t] = \{k' \in \mathcal{P}_{m_i}[t] | k \succ_{m_i} k'\}$;
15: **else**
16: $R'_{m_i}[t] = \{k \in \mathcal{K} | \mu[t](m_i) \succ_{m_i} k\}$;
17: $\mathcal{L}_{m_i}[t] = \{R_{m_i}[t]\} \cup \{R'_{m_i}[t]\}$;
18: **for** $l \in \mathcal{L}_{m_i}[t]$ **do**
19: $\mathcal{P}_l[t] \leftarrow \mathcal{P}_l[t] \setminus \{m_i\}$;
20: $\mathcal{P}_{m_i}[t] \leftarrow \mathcal{P}_{m_i}[t] \setminus \{l\}$;
21: **until** $\mu[t] = \mu[t-1]$
22: ***Phase 3: BS Assignment***: $\mu \mapsto \mathbf{A}$;

2. The optimality property in the proposed matching game only holds for the proposing side, i.e., BSs.

Theorem 3. *Algorithm 2 produces a weak Pareto Optimal (PO) solution for the problem presented in (23).*

Proof. Please see the Appendix E, available in the online supplemental material □

5.4 Computation Complexity and Practical Implementation

For each MVNO user m_i, the complexity of building the preference profile using any standard sorting algorithm is $\mathcal{O}(|\mathcal{K}|\log(|\mathcal{K}|))$ and similarly, the complexity of building the preference profile at BS k for all MVNO users is $\mathcal{O}(|\mathcal{U}|\log(|\mathcal{U}|))$. So, the input to Algorithm 2 is $\xi = \sum_{k\in\mathcal{K}} |\mathcal{P}_k| + \sum_{m_i\in\mathcal{U}} |\mathcal{P}_{m_i}| = 2|\mathcal{K}||\mathcal{U}|$, where $|\mathcal{P}|$ denotes the length of preference profile $\mathcal{P}$. As Algorithm 2 terminates after a finite number of iterations (the convergence is stated in Property 1), it can be seen that in the worst case, the time complexity of Algorithm 2 is quadratic (i.e., $\mathcal{O}(\xi) = \mathcal{O}(|\mathcal{K}||\mathcal{U}|) = \mathcal{O}(\max(|\mathcal{K}|, |\mathcal{U}|)^2)$), which is practically acceptable.

6 In-slice Subchannel Assignment and Energy Efficiency Optimization

In this section, we first analyze the in-slice subchannel assignment and energy efficiency optimization problem for a given BS assignment solution. We then design a distributed algorithm that can be implemented at each BS to assign resources (i.e., subchannels) to its associated user to achieve the design objectives i.e., service contract agreement and energy-efficiency for the InP.

For a given pricing decision and BS assignment solution, i.e., $\{\mathbf{p}^*, \mathbf{A}^*\}$, the subchannel assignment for the energy efficiency optimization problem can be reformulated as follows:

$$
\begin{aligned}
\max_{(\mathbf{Y},\mathbf{P})} \quad & V\frac{\sum_{k\in\mathcal{K}}\sum_{m_i\in\mathcal{S}_k}\sum_{c\in\mathcal{C}} y_{m,i}^{(c)} R_{k,m,i}^{(c)}(\mathbf{P}^{(c)})}{\sum_{k\in\mathcal{K}}(\sum_{m_i\in\mathcal{S}_k}\sum_{c\in\mathcal{C}} y_{m,i}^{(c)} P_k^{(c)} + P_k^0)} \\
& + \sum_{k\in\mathcal{K}}\sum_{m_i\in\mathcal{S}_k}\sum_{c\in\mathcal{C}} Q_{m,i} y_{m,i}^{(c)} R_{k,m,i}^{(c)}(\mathbf{P}^{(c)}) \\
\text{s.t.} \quad & C_2, C_3, \\
& C_5'': \sum_{m_i\in\mathcal{S}_k}\sum_{c\in\mathcal{C}} y_{m,i}^{(c)} R_{k,m,i}^{(c)}(\mathbf{P}^{(c)}) \le B_k, \ \forall k\in\mathcal{K}, \\
& C_6'': \sum_{m_i\in\mathcal{S}_k}\sum_{c\in\mathcal{C}} y_{m,i}^{(c)} P_k^{(c)} \le P_k^{\max}, \ \forall k\in\mathcal{K}, \\
& C_7'': y_{m,i}^{(c)} \in \{0,1\}, P_k^{(c)} \ge 0, \ \forall m,c,k.
\end{aligned} \tag{26}
$$

We see that the objective function in (26) is a ratio of two functions, which generally results in a non-convex function. We note that there is no standard approach for solving non-convex optimization problems. Therefore, in order to derive an efficient RA algorithm for the considered problem, we introduce the following transformation.

6.1 Transformation of the Objective Function

The optimization problem (26) is classified as a nonlinear fractional program [20]. We define $\mathcal{F}$ as the set of feasible solutions of the optimization problem (26) and define the maximum energy efficiency q^* of the considered InP as

$$q^* = \frac{\mathcal{R}(\mathbf{Y}^*, \mathbf{P}^*)}{\mathcal{P}(\mathbf{Y}^*, \mathbf{P}^*)} = \max_{(\mathbf{Y},\mathbf{P})} \frac{\mathcal{R}(\mathbf{Y}, \mathbf{P})}{\mathcal{P}(\mathbf{Y}, \mathbf{P})}, \ \forall \{\mathbf{Y}, \mathbf{P}\} \in \mathcal{F}. \tag{27}$$

Theorem 4 (Problem Equivalence). *The maximum energy efficiency q^* is achieved if and only if*

$$\max_{(\mathbf{Y},\mathbf{P})} \mathcal{R}(\mathbf{Y}, \mathbf{P}) - q^*\mathcal{P}(\mathbf{Y}, \mathbf{P}) = \mathcal{R}(\mathbf{Y}^*, \mathbf{P}^*) - q^*\mathcal{P}(\mathbf{Y}^*, \mathbf{P}^*) = 0, \tag{28}$$

for $\mathcal{R}(\mathbf{Y}, \mathbf{P}) \ge 0$ and $\mathcal{P}(\mathbf{Y}, \mathbf{P}) > 0$.

Proof. Please refer to [20] for a proof of Theorem 4. □

By Theorem 4, for any optimization problem with an objective function in fractional form, there exists an equivalent objective function in subtractive form, e.g., $\mathcal{R}(\mathbf{Y}, \mathbf{P}) - q^*\mathcal{P}(\mathbf{Y}, \mathbf{P})$. As a result, we can focus on the equivalent objective function to solve the subproblem of subchannel assignment and power allocation for MVNOs' users.

6.2 Iterative Algorithm for Energy Efficiency Maximization

In this section, we propose an iterative algorithm (known as the Dinkelbach method [20], [21]) for solving (26) with an equivalent objective function. The proposed algorithm is summarized in Algorithm 3 and the convergence to the optimal energy efficiency is guaranteed if we are able to solve the inner problem (29) in each iteration. Please refer to [20] or [21] for a proof of the convergence of Algorithm 3.

Algorithm 3. Energy Efficiency Maximization Algorithm

Initialization: T_{max}, ϵ, $\tau = 0$, $q[0] = 0$, and Flag = **false**;
repeat
  Solve problem (29) for a given q and obtain $\{\mathbf{Y}[\tau], \mathbf{P}[\tau]\}$;
  if $\mathcal{R}(\mathbf{Y}[\tau], \mathbf{P}[\tau]) - q[\tau]\mathcal{P}(\mathbf{Y}[\tau], \mathbf{P}[\tau]) < \epsilon$ **then**
    Flag = **true**;
    return $\{\mathbf{Y}^*, \mathbf{P}^*\} = \{\mathbf{Y}[\tau], \mathbf{P}[\tau]\}$
    $q^* = \frac{\mathcal{R}(\mathbf{Y}[\tau], \mathbf{P}[\tau])}{\mathcal{P}(\mathbf{Y}[\tau], \mathbf{P}[\tau])}$;
  else
    $q[\tau+1] = \frac{\mathcal{R}(\mathbf{Y}[\tau], \mathbf{P}[\tau])}{\mathcal{P}(\mathbf{Y}[\tau], \mathbf{P}[\tau])}$ and $\tau = \tau + 1$;
    Flag = **false**;
until Flag = **true** or $\tau = T_{max}$;

As presented in Algorithm 3, in each iteration of the main loop, we solve the following optimization problem for a given parameter q

$$
\begin{aligned}
\max_{(\mathbf{Y},\mathbf{P})} \quad & \mathcal{R}'(\mathbf{Y}, \mathbf{P}) - q\mathcal{P}(\mathbf{Y}, \mathbf{P}) \\
\text{s.t.} \quad & C_2, C_3, C_5'', C_6'', C_7'',
\end{aligned} \tag{29}
$$

where

$$\mathcal{R}'(\mathbf{Y}, \mathbf{P}) = \sum_{k\in\mathcal{K}}\sum_{m_i\in\mathcal{S}_k}\sum_{c\in\mathcal{C}} (V + Q_{m,i}) y_{m,i}^{(c)} R_{k,m,i}^{(c)}(\mathbf{P}^{(c)}).$$

The transformed problem (29) is a mixed combinatorial and non-convex optimization problem whose non-convex nature comes from the power allocation variables. The cross-tier and co-tier interference appearing in the denominator of the SINR expression couples the power allocation variables. On the other hand, the combinatorial nature comes from the integer constraint for subchannel assignment. We propose a procedure to solve (29) as summarized in Algorithm 4.

The procedure starts by initializing the transmit power of BS k on all subchannels c equally, i.e., $P_k^{(c)}[0] = P_k^{\max}/C$. For fixed values of $P_k^{(c)}[t]$, problem (29) becomes a standard integer linear program that can be solved efficiently by using a standard solver (line 3). Assume that we have a subchannel assignment policy to determine $\mathbf{Y}[t]$; problem (29) is still a non-convex problem and difficult to solve directly (line 4). We now propose a solution for the non-convex power allocation problem based on DC programming [23], [24].

Algorithm 4. Procedure to Solve (29)

1: Initialize $P_k^{(c)}[0] = P_k^{\max}/C$, and iteration $t = 0$;
2: **repeat**
3: Solve problem (29) for a fixed $P_k^{(c)}[t]$ and obtain $\mathbf{Y}[t]$;
4: Update $P_k^{(c)}[t+1]$ by solving (29) for given $\mathbf{Y}[t]$;
5: Set $t = t + 1$;
6: **until** Convergence of $\mathbf{P}$ and $\mathbf{Y}$;

Theorem 5. *For a feasible problem (29), Algorithm 4 will converge to a local maximum of (29).*

Proof. Please see a similar proof in [24]. □

6.3 D.C. Based Power Allocation

Apparently, for a given subchannel assignment policy $\mathbf{Y}[t]$, problem (29) is not convex because the rate function in (5) is non-concave with respect to the variable $\mathbf{P}$. To overcome such a major difficulty, we adopt the following successive convex approximation approach [23], [24] and find the optimal power allocation in another time scale t_p.

We first express the rate function (5) in D.C. form as

$$\sum_{c\in\mathcal{C}} y_{m,i}^{(c)} R_{k,m,i}^{(c)}(\mathbf{P}^{(c)}) = f_k(\mathbf{P}) - g_k(\mathbf{P}), \tag{30}$$

where $f_k(\mathbf{P})$ and $g_k(\mathbf{P})$ are the two concave functions defined as follows:

$$f_k(\mathbf{P}) = \sum_{c\in\mathcal{C}} y_{m,i}^{(c)} \ln\left(\sigma_{m,i}^{(c)} + \sum_{j\in\mathcal{K}} h_{j,m,i}^{(c)} P_j^{(c)}\right), \tag{31}$$

$$g_k(\mathbf{P}) = \sum_{c\in\mathcal{C}} y_{m,i}^{(c)} \ln\left(\sigma_{m,i}^{(c)} + \sum_{j\in\mathcal{K}, j\neq k} h_{j,m,i}^{(c)} P_j^{(c)}\right). \tag{32}$$

Then, we employ the approximation [23]

$$g_k(\mathbf{P}) \approx g_k(\mathbf{P}[t_p-1]) + \nabla g_k^T(\mathbf{P}[t_p-1])(\mathbf{P} - \mathbf{P}[t_p-1]), \tag{33}$$

for the power $\mathbf{P}[t_p-1]$ from iteration $t_p - 1 \geq 0$. Here, the gradient $\nabla g_k^T(\mathbf{P})$ is a vector of length $(K+1)C$ whose elements are defined as

$$\nabla g_k(\mathbf{P})^{(Cj+c)} \triangleq \begin{cases} 0, & \text{if } j = k, \\ \dfrac{y_{m,i}^{(c)} h_{j,m,i}^{(c)}}{\sum_{l\in\mathcal{K}\setminus\{k\}} h_{l,m,i}^{(c)} P_l^{(c)} + \sigma_{m,i}^{(c)}}, & \text{if } j \in \mathcal{K}\setminus\{k\}, \end{cases} \tag{34}$$

for $c \in \mathcal{C}$. From (30) and (33), we have

$$\begin{aligned} \sum_{c\in\mathcal{C}} R_{k,m,i}^{(c)}(\mathbf{P}^{(c)}) \approx f_k(\mathbf{P}) - g_k(\mathbf{P}[t_p-1]) \\ - \nabla g_k^T(\mathbf{P}[t_p-1])(\mathbf{P} - \mathbf{P}[t_p-1]). \end{aligned} \tag{35}$$

The right-hand side of this equation is actually a concave function with respect to $\mathbf{P}$.

The approximation (35) allows us to recast the non-convex problem (29) into a sequence of convex optimization subproblems with respect to $\mathbf{P}$. Starting from a feasible $\mathbf{P}[0]$, the optimal solution $\mathbf{P}[t_p]$ at iteration $t_p > 0$ is determined upon solving the following convex program:

$$\begin{aligned} \max_{\mathbf{P}} \; & \sum_{k\in\mathcal{K}} \sum_{m_i\in\mathcal{S}_k} \sum_{c\in\mathcal{C}} (V + Q_{m,i}) R_{k,m,i}^{(c)}(\mathbf{P}^{(c)}) \\ & - q \sum_{k\in\mathcal{K}} \sum_{m_i\in\mathcal{S}_k} \sum_{c\in\mathcal{C}} \left(y_{m,i}^{(c)} P_k^{(c)} + P_k^0 \right) \\ \text{s.t.} \; & \sum_{m_i\in\mathcal{S}_k} \sum_{c\in\mathcal{C}} R_{k,m,i}^{(c)}(\mathbf{P}^{(c)}) \leq B_k, \; \forall k \in \mathcal{K}, \\ & \sum_{m_i\in\mathcal{S}_k} \sum_{c\in\mathcal{C}} y_{m,i}^{(c)} P_k^{(c)} \leq P_k^{\max}, \; \forall k \in \mathcal{K}, \\ & 0 \geq P_k^{(c)} \geq 0, \; \forall k \in \mathcal{K}, c \in \mathcal{C}. \end{aligned} \tag{36}$$

Based on the KKT condition and dual method [25], the optimal transmit power of BSs and the Lagrange multipliers can be obtained as in (37), (38), and (39), respectively.

$$\nu_k[t_s+1] = \left[\nu_k[t_s] + \kappa_\nu \left(\sum_{m_i\in\mathcal{S}_k} \sum_{c\in\mathcal{C}} y_{m,i}^{(c)} P_k^{(c)}[t_s] - P_k^{\max} \right) \right]^+. \tag{39}$$

From the above derivations, we propose a distributed power allocation algorithm as described in Algorithm 5.

Remark 3.

(1) Interference terms $\mathcal{I}_{k,m,i}^{(c)}(\mathbf{P}_{-k}^{(c)})$ locally measured and fed back by UE $m_i \in \mathcal{S}_k$, where we define

$$\mathcal{I}_{k,m,i}^{(c)}(\mathbf{P}_{-k}^{(c)}) \triangleq \sum_{l\in\mathcal{K}\setminus\{k\}} h_{l,m,i}^{(c)} P_l^{(c)} + \sigma_{m,i}^{(c)}. \tag{40}$$

(2) Aggregate interference terms $\mathcal{Q}_{k,m,i}^{(c)}(\mathbf{P}^{(c)})$ broadcast by BS $k \in \mathcal{K}$, where we define

$$\mathcal{Q}_{k,m,i}^{(c)}(\mathbf{P}^{(c)}) \triangleq \mathcal{I}_{k,m,i}^{(c)}(\mathbf{P}_{-k}^{(c)}) + h_{k,m,i}^{(c)} P_k^{(c)}. \tag{41}$$

$$P_k^{(c)}[t_s+1] = \left[\frac{\sum_{m_i\in\mathcal{S}_k} (V + Q_{m,i})(1 + \mu_k[t_s]) y_{m,i}^{(c)} h_{k,m,i}^{(c)} P_k^{(c)}[t_s] \left(\frac{1}{\mathcal{Q}_{k,m,i}^{(c)}(\mathbf{P}^{(c)}[t_s])} \right)}{(q + \nu_k[t_s]) \sum_{m_i\in\mathcal{S}_k} y_{m,i}^{(c)} + \sum_{j\in\mathcal{K}\setminus\{k\}} \sum_{n_i\in\mathcal{S}_j} (V + Q_{n,i})(1 + \mu_j[t_s]) y_{n,i}^{(c)} h_{k,n,i}^{(c)} \left(\frac{1}{\mathcal{I}_{j,n,i}^{(c)}(\mathbf{P}_{-j}^{(c)}[t_p-1])} \right)} \right]_0^{P_k^{\max}} \tag{37}$$

$$\mu_k[t_s+1] = \left[\mu_k[t_s] + \kappa_\mu \left\{ \sum_{m_i\in\mathcal{S}_k} \sum_{c\in\mathcal{C}} y_{m,i}^{(c)} \ln\left(\frac{\mathcal{Q}_{k,m,i}^{(c)}(\mathbf{P}^{(c)}[t_s])}{\mathcal{I}_{k,m,i}^{(c)}(\mathbf{P}_{-k}^{(c)})[t_p-1]} \right) - \frac{y_{m,i}^{(c)} \mathcal{I}_{k,m,i}^{(c)}(\mathbf{P}_{-k}^{(c)}[t_s])}{\mathcal{I}_{k,m,i}^{(c)}(\mathbf{P}_{-k}^{(c)})[t_p-1]} + |\mathcal{C}| - B_k \right\} \right]^+ \tag{38}$$

(3) Channel gains $h_{k,m,i}^{(c)}$ measured and fed back by UE $m_i \in \mathcal{S}_k$. We assume a block fading model, channel information only needs to be sent once at the beginning of the Algorithm 5 [24].

Algorithm 5. Distributed Power Allocation at BS $k \in \mathcal{K}$

```
Initialize: {μ_k[0], ν_k[0]} > 0, ∀c ∈ C, ∀m_i ∈ S_k, P_k^(c)[0] = 0,
    P̃_k^(c)[0] = 0, t_p = 0 and t_s = 0;
repeat
    repeat
        User m_i(c) ∈ S_k measures and reports h_{k,m,i}^(c) and
            I_{k,m,i}^(c)(P̃_{-k}^(c)[t_s]) to BS k ∈ K;
        Compute Q_{k,m,i}^(c)(P̃^(c)[t_s]) ∀c ∈ C and broadcasts to
            other BSs j ∈ K\{k};
        Compute P̃_k^(c) by (37) ∀c ∈ C;
        Update μ_k, ν_k according to (38) and (39), respectively;
        Set t_s := t_s + 1;
    until μ_k, ν_k converge
    Set P_(k)[t_p] = P̃_(k)[t_s];
    Broadcast I_{k,m,i}^(c)(P_{-k}^(c)[t_p]) to other BSs j ∈ K\{k};
    Set t_p := t_p + 1;
until P_(k) converge;
```

Theorem 6. *For a given subchannel assignment policy* $\mathbf{Y}[t]$*, Algorithm 5 converges to a locally optimal solution* $\mathbf{P}[t]$ *of problem (29).*

Proof. Please see Appendix F, available in the online supplemental material. □

7 Price Decision Optimization

In the independent MVNOs model, the price charged to the MVNOs will be determined according to the time average throughput of the MVNOs' users (or expected throughput from serving the MVNOs' users). The short-term price decision optimization problem subject to stability of the contract agreement queue can be formulated as follows:

$$\max_{\boldsymbol{\beta}(t)\in\Omega} \sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{U}_m}\left[V\beta_m(t)r_{m,i}(t) - Q_{m,i}(t)r_{m,i}(t)\right], \tag{42}$$

where $\Omega = \{\beta_m, m \in \mathcal{M} | \beta^{\min} \le \beta_m \le \beta^{\max}\}$, and $r_{m,i}(t)$ from (9). The first term of the objective function is the revenue of the InP from serving MVNOs' users, the second term represents the penalty for the contract agreement queue backlog. By maximizing this objective function, the InP can optimize its revenue and push the virtual queue backlog down to maintain the stability of the system and guarantee the service contract agreement with the MVNOs. Note that in case of the competing MVNOs model for which the users' rates are derived in Section 3.2, we do not need to solve any price decision optimization problem. The time-average throughput of user i of MVNO m for each pricing period can be expressed as

$$\mathbb{E}(R_{m,i}(t)) = \int_0^{R_{m,i}^{\max}} R_{m,i}f(R_{m,i})dR_{m,i}, \tag{43}$$

where $R_{m,i}^{\max}$ is the upper of the transmission rate of user i of MVNO m up to time t, respectively, and $f(R_{m,i})$ is the probability density function of $R_{m,i}$. The transmission rate depends on SINR, which depends on distributions of all power channel gains (follows exponential variables since the amplitude of channel gain follows a Rayleigh distribution).

Due to the operator $[\cdot]_x$ of $r_{m,i}(t)$ in (9), problem (42) is nonconvex. For convenience, we define a new variable

$$z_m(t) = \begin{cases} 1, & \text{if } \beta_m(t) < \frac{\mathfrak{B}_m^{\max}}{|\mathcal{U}_m| r_{m,i}^{\min}}, \\ 0, & \text{otherwise.} \end{cases} \tag{44}$$

Then, $r_{m,i}(t) = \mathfrak{B}_m^{\max}/|\mathcal{U}_m|\beta_m(t)$ when $z_m(t) = 1$, and $r_{m,i}(t) = r_{m,i}^{\min}$ when $z_m(t) = 0$. Without loss of generality, we assume that all users belonging to the same MVNO have the homogeneous minimum data rate requirement, i.e., $r_{m,i}^{\min} = r_m^{\min}, \forall i \in \mathcal{U}_m$. Then, the revenue of the InP at each time t is equivalent to

$$\mathcal{R}_m(\beta_m(t), z_m(t)) = z_m(t)\frac{\mathfrak{B}_m^{\max}}{|\mathcal{U}_m|} + (1 - z_m(t))\beta_m(t)r_m^{\min}. \tag{45}$$

The price at time t can be obtained by solving the following problem:

$$\max_{(\boldsymbol{\beta},\mathbf{z})}. \sum_{m\in\mathcal{M}}\left[V\mathcal{R}_m(\beta_m(t), z_m(t)) - \sum_{i\in\mathcal{U}_m} Q_{m,i}(t)r_{m,i}(t)\right], \tag{46}$$

where V is a controlled parameter.

We see that problem (46) is a mixed-boolean program, which requires exponential computation efforts to obtain the optimal solution through exhaustive search. This motivates us to propose an efficient algorithm in Algorithm 6 to solve this problem.

Algorithm 6 is developed based on the BnB method [31], which enables us to find the global optimal solution for price decision problem (42). Let $\mathcal{Q}_{init} = \{\boldsymbol{\beta}, \mathbf{z}\}$ be the original search space, including all possible combinations of indicator variable $\mathbf{z}$. The proposed algorithm maintains a set of subdomains $\boldsymbol{Q} = \{\mathcal{Q}_t \subset \mathcal{Q}_{init}, t = 1, 2, \ldots\}$, where t represents the iteration of the algorithm. For any $\mathcal{Q}_t$, let $\Phi_{ub}(\cdot)$ and $\Phi_{lb}(\cdot)$ denote the upper and lower bounds. We refer to $\Phi_{ub}(\mathcal{Q}_t)$ and $\Phi_{lb}(\mathcal{Q}_t)$ as the local upper and local lower bounds, respectively, which correspond to subdomain $\mathcal{Q}_t$.

Algorithm 6. Price Decision Algorithm

```
initialize: t = 0; Q = {Q_init}; L_0 = Φ_lb(Q_init); U_0 = Φ_ub(Q_init);
    ε > 0;
repeat
    t ← t + 1;
    Q_t = {Q ∈ Q | m = argmin(|z*_m − 1/2|)};
    Q_t^(0) = {β, z | z_m = 0}; Q_t^(1) = {β, z | z_m = 1};
    Q = {Q \ Q_t} ∪ {Q_t^(0), Q_t^(1)};
    for Q_t^(i), i ∈ {0, 1} do
        Calculate Φ_lb(Q_t^(i)) and Φ_ub(Q_t^(i));
    U_t = min(U_t, Φ_ub(Q_t^(i)), i = 0, 1);
    L_t = min(L_t, Φ_lb(Q_t^(i)), i = 0, 1);
    Q^(pru) = {Q ∈ Q | Φ_lb(Q) ≥ U_t)};
    Q = {Q \ Q^(pru)};
until U_t − L_t ≤ ε;
```

The algorithm starts by relaxing the boolean variable z_m i.e., $0 \le z_m \le 1, \forall m \in \mathcal{M}$, and then solves the relaxed problem to obtain a lower bound $L_0 = \Phi_{lb}(\mathcal{Q}_{init})$ for the original

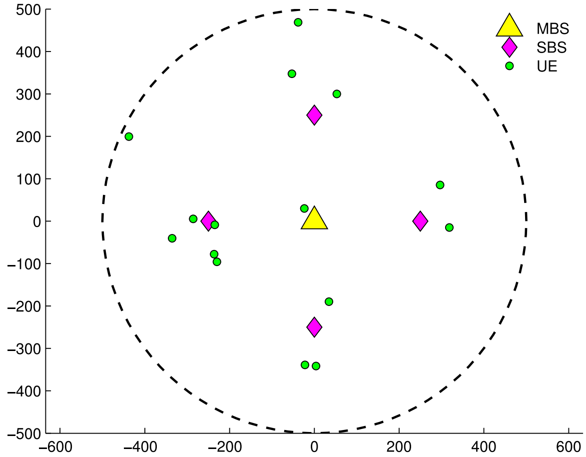


Fig. 2. Network topology.

problem (46). Then, we round the optimal relaxed variables z_m^* to 0 or 1, $\forall m \in \mathcal{M}$, and solve (46) again with these fixed values of z_m^* to obtain the upper bound $U_0 = \Phi_{ub}(\mathcal{Q}_{init})$ (line 2). At each iteration t, we split the search space into two subspaces $\mathcal{Q}_t^{(0)}$ and $\mathcal{Q}_t^{(1)}$ by picking $\mathcal{Q}_t \in Q$ such that $m = \text{argmin}(|z_m^* - 1/2|)$, then update Q by removing $\mathcal{Q}_t$ (lines 5-7). We then calculate the lower and upper bounds for each subspace and choose the one with the smallest lower bound (lines 8-12). Finally, the subspaces that satisfy $\Phi_{lb}(\mathcal{Q}) \geq U_t$ are removed, since every point in such a space leads to a performance lower than the current upper bound (lines 13-14). If $U_t - L_t \leq \epsilon$, the algorithm terminates.

Lemma 3. *The Algorithm 6 converges to the optimal solution of price decision problem (42).*

Proof. The proof is similar to the one provided in [31]. □

8 Simulation Results

8.1 Simulation Setting

In this section, we evaluate the system performance of the proposed framework using simulations. The network scenario is shown in Fig. 2, where the MVNOs' subscribed users are randomly located inside a macro cell of radius 500 meters covering 4 SBSs with a radius of 100 meters each. These MBS and SBSs belong to the InP. We consider multiple MVNOs with number of users can be varied. The distance between each SBS and the MBS is 250 meters, whereas the shortest distance from an SBS to another SBS is $250\sqrt{2}$ meters. We assume the InP owns $|\mathcal{C}| = 10$ OFDMA subchannels, each of which has a total bandwidth of 180 KHz. We set $P_0^{\max} = 40$ Watt (W) and $P_k^{\max} = 4$ W, $\forall m \in \mathcal{M} \backslash \{0\}$. The noise power is assumed to be 10^{-13} W for all subchannels $c \in \mathcal{C}$. The small-scale fading coefficients of the BS-to-user links are generated as i.i.d. Rayleigh random variables with unit variance. Channel gains are set as $h_{k,m,i}^{(c)} = \chi^{(c)} d_{k,m,i}^{-\alpha}$, where $\chi^{(c)}$ is a random value generated according to the Rayleigh distribution, $d_{k,m,i}$ is the geographical distance between BS k and user i of MVNO m, and $\alpha = 3$ is the pathloss exponent. We set the static circuit power consumption as $P_0^{fix} = 40$ W and $P_k^0 = 4$ W for the MBS and SBSs, respectively.

For the backhaul connections, we adopt the specifications of a commercial optical fiber modem [22], [34] which supports three different data rates for backhaul within a distance of 2.5 km: $R_1 = 11.184$ Mbps, $R_2 = 34.368$ Mbps, and $R_3 = 44.736$ Mbps. We fix the backhaul capacity of MBS to $B_0 = R_3$ while varying the backhaul capacity of SBSs between R_1, R_2 and R_3 to analyze the performance of our proposed framework under different backhaul capacities. For the backhaul link topology, we adopt a star backhaul link topology, in which all BSs connect directly to the central controller via individual backhaul links. The MVNOs' minimum data rate is assumed to be varied between $r_{\min} = 5, 8,$ and 10 bps/Hz/user. The price bounds are set to be $\beta_{\min} = 0.1$ and $\beta_{\max} = 0.3$ and the price scaling factor θ is set to be 2.

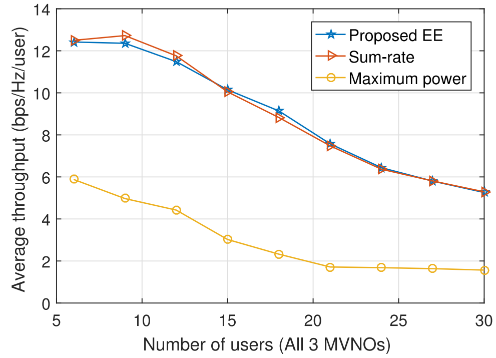


Fig. 3. Average throughput (bps/Hz/user).

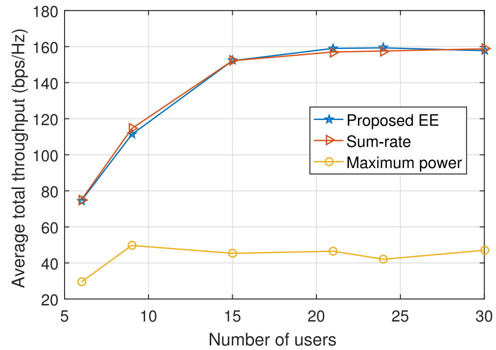


Fig. 4. Average total network throughput (bps/Hz).

To evaluate the energy efficiency performance of our proposed framework (denoted by "Proposed EE"), two algorithms are presented as baselines for comparisons. The first baseline is the maximum power allocation (denoted by "Maximum power"), i.e., the equal and fixed transmit power $P_k^{\max}/|\mathcal{S}_k|$ is set for different subchannels, and the optimal power allocation derived in (37) is not utilized. The second baseline is maximizing the network sum-rate, i.e., the numerator of η_{EE} in (7) (denoted by "Sum-rate") with the same constraints in (26).

8.2 Numerical Results

Fig. 3, it can be observed that the average throughput of each user decreases with the increase in the number of users in the network. The decrease in throughput of each user is due to the sharing of limited resources among the larger number of users and enhanced network interference. Moreover, in Fig. 4, it can be seen that, the average total throughput increases with the increase in the number of users in the network, until it reaches a saturation point as the number of users and/or the number of MVNOs become sufficiently large, i.e., after 20 and above network users. The throughput saturates due to limited resources available in the network, i.e., for the current simulation setting. Furthermore, we observe that the

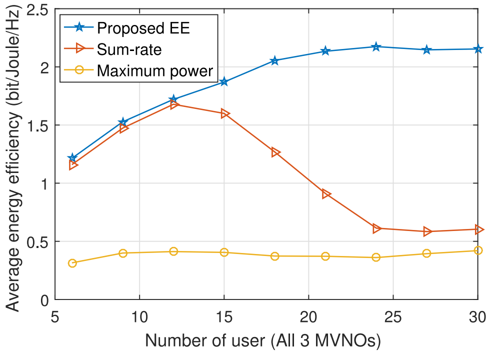


Fig. 5. Average energy efficiency (bit/Joule/Hz).

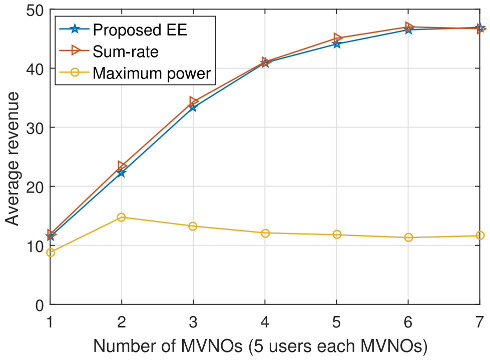


Fig. 6. Average revenue.

proposed scheme achieves indistinguishable throughput when compared to the sum-rate baseline and achieves a significant performance gain when compared to the maximum power baseline scheme.

In Fig. 5, we evaluate the average energy efficiency by increasing the number of users in the network. The energy efficiency also increases with the increase in the number of network users. The main reason behind this is as the number of users are increasing in the network, the underutilized resources are getting occupied, then, the energy efficiency also increases. However, energy efficiency also saturates and reaches its peak as all the network resources get utilized, i.e., after 20 and above network users. Moreover, for both baselines as the number of users in the network increase, the power consumption also increase which reduces the energy efficiency.

Moreover, in Fig. 6, we observe a similar trend in terms of average revenue gain, i.e., once the saturation point of total network throughput is achieved, the revenue also saturates. Note, that the revenue depends upon the achievable data rate, then, similar revenue gains are achieved by both the proposed and sum-rate scheme whereas low revenue is achieved due to low achievable rate for the maximum power scheme.

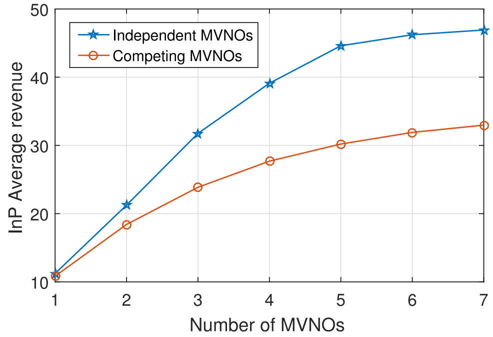
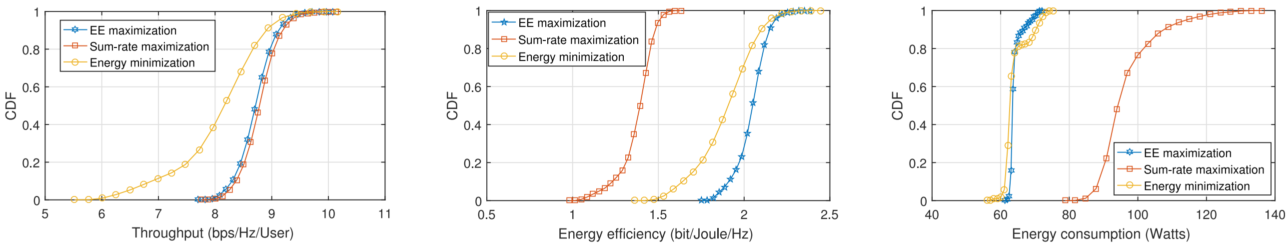


Fig. 7. Independent MVNOs versus Competing MVNOs.

We have performed additional simulation to compare the InP's revenue in two case of MVNOs model: independent MVNOs model (denoted by "Independent MVNOs") and competition as a non-cooperative game between MVNOs model (denoted by "Competing MVNOs"). We have normalized the parameters in the leasing price function in competing MVNOs model such that the revenue of the InP in case of one MVNO in both models is similar. In Fig. 7, it can observed that, the revenue of the InP in case of no interaction between MVNOs is higher than in case there is competition among MVNOs. The intuition is that when the InP independently negotiates with each MVNO the InP will achieve higher revenue than when all MVNOs compete with each others.

Next, we perform simulations and compare the performances under different design objectives, namely, maximizing energy efficiency (i.e., our proposed model denoted by "EE maximization") and minimizing energy consumption. The baseline minimizing energy consumption scheme corresponds to the same design, however, we replace the objective of maximizing the energy efficiency with minimizing energy cost of all BSs, i.e., minimizing energy consumption (denoted by "Energy minimization"). Moreover, we also compare the results with the the sum-rate maximization baseline (denoted as "Sum-rate maximization").

In Figs. 8a and 8c, it can be seen that our proposed "EE maximization" design can achieve low energy consumption similar the baseline "Energy minimization" scheme, while achieve similar throughput compared to sum-rate maximization baseline. Moreover, a significant gain in terms of average throughput can be observed with respect to the baseline "Energy minimization" scheme (Fig. 8a). Thus, achieving high throughput gain and low energy consumption, our proposed "EE maximization" scheme can achieve the highest energy efficiency in comparison to both the baselines, i.e., "Sum-rate maximization" and "Energy minimization", by 46 and 12 percent, respectively (Fig. 8b).



(a) Throughput (b) Energy efficiency (c) Energy consumption

Fig. 8. Energy efficiency versus energy consumption.

9 CONCLUSION

In this paper, we have proposed a dynamic resource assignment and pricing scheme for wireless network virtualization which can be applied for current mobile market to eliminate resource underutilization and unfairness due to the static resource assignment and fixed pricing scheme. We have analyzed problem of multiple MVNOs and InP and proposed distributed practical algorithms to achieve the objective design in terms of revenue and energy efficiency. The proposed algorithms can overcome the difficulty of computation and practice implementation and can be implemented distributively at each network entities, i.e., server, base stations and users. Through extensive simulations, we show that our approaches outperform traditional designs, i.e., sum-rate maximization, transmit power maximization, and energy cost minimization, in terms of throughput, energy efficiency, energy consumption and revenue. For future direction, we will adopt non-orthogonal multiple access (NOMA) technique in wireless network virtualization for massive connectivity with limited spectrum resource.

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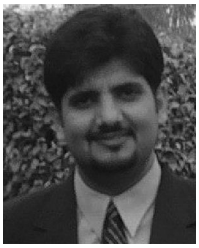

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