Multi-stage Stackelberg Game Approach for Colocation Datacenter Demand Response

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***Abstract*—There have been many recent studies on the Demand Response (DR) of Datacenters (DCs). Nonetheless, (i) DR of Colocation Datacenters (CDCs), and (ii) the role of Demand Response Provider (CSP) have been largely overlooked. CDCs differ from big owner-operated DCs in that the operator has no control over their tenants, and thus, requiring a mechanism for the operator to give tenants incentives to reduce their electricity usage. CSP uses compensation price as a guidance for customers' response. To fill the gap, we propose an incentive mechanism for CDC DR that studies the interaction between the CSP, CDCs and tenants. Firstly, the strategic behaviors of these interactions are formulated as a three-stage Stackelberg game which contains a separate problem at each stage. In Stage I, the CSP solves an optimal compensation pricing problem. In Stage II, each CDC operator finds its own optimal procurement and reward strategy. In Stage III, the optimal tenants' energy reduction is calculated. Secondly, we examine both exact and approximate solution at Stage II, and propose an efficient algorithm to obtain the optimal CSP price in Stage I. Finally, the extensive numerical analysis (a) shows that the CLT-based approximation achieves similar solutions compared to the exact analysis, and (b) illustrates the comparisons between the optimal CSP individual cost and the social cost.**

I. Introduction

Data-centers (DCs) have large yet flexibility power demands [1], and hence, are ideal participants for demand response (DR) programs to enhance power grid reliability [2]. Although large owner-operated DCs (e.g., Google) have garnered interests for DR [3], an important segment of DCs has been left under-explored *colocation data-centers* or CDCs (e.g., Equinix). Recently, there have been limited studies on DR for CDCs [4]–[6]. However, the full significance of CDCs remains to be discovered: First, CDCs could be a common solution to a variety of Internet content service providers, (e.g., Twitter, Akamai) and cloud service providers [7]. Second, demands for CDCs have been rapidly increasing: There are more than 1200 CDCs in the U.S. alone [8]. Moreover, CDC market is forecast to reach $43 billion in 2018 [9]. In addition to its economic importance, CDC is also ideal for DR: First, CDCs consume a huge amount of power, e.g., CDCs use 37% of electricity of DCs power usages in the U.S [10]. Second, many CDCs are in densely populated urban areas, e.g., Los Angeles [8], where DR is often necessary.

Different from large owner-operated DCs, a CDC houses multiple tenants who individually manage servers in a shared building. The building operator usually supports the facility (e.g., cooling system) without any control over tenants IT systems. Thus, there are growing interests on incentive mechanisms for tenants to cooperate with operator for DR. DR can be categorized into mandatory *emergency* DR and voluntary *economic* DR [11]. Hence, the parameters such as incentive payment and costs, are considered constants, effectively ignoring the role of the Curtailment Service Provider (CSP) [5], [6]. CSP is an authorized mediator between independent system operator (ISO) and the CDCs customers for participating in the DR program.

In economic DR, an ISO requests its customers a DR during peak power periods with high electricity prices in exchange for monetary compensations via CSP. Each customer has an *elastic* DR capacity and can curtail the energy consumption to receive payment without sacrificing their comfort. Since regulations impose an average rate on customers' usage charges, CSP emulates the whole sale price pattern [12] and coordinates the reaction of the customers to the CSP payment price. Existing works on CDC economic DR [4], [13], neglect the significance of CSP. Hence, we fill this gap with an incentive mechanism for CDC economic DR under the CSP control.

In summary, the contributions of our study are as follows: We design an incentive mechanism to incentivize CDC to curtail energy consumption for economic DR. Strategic decisions of CSP, CDCs and tenants and their interactions are modeled as a three-stage Stackelberg game in which a Stackelberg equilibrium where CSP, CDCs and tenants make optimal decisions. In Stage II, we design a near-optimal approximate approach that only requires limited information. In Stage I, we design an efficient algorithm by reducing the search space of the CSP's optimal pricing problem. We perform an extensive numerical analysis for finding the optimal compensation price, and compare the individual CSP cost with social CSP cost to measure the effect of an individual strategy on the social welfare.

Most works on DR of DC, [3], [14]–[16], studied the ancillary and/or emergency DR. In [17], the authors studied the pricing mechanism of DCs for economic DR. As CDC

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operator has no control over tenants' IT systems, most of the mentioned approaches cannot be directly applied to CDCs. The earliest study on CDC economic DR, [4], relies on the tenants' best-effort, which can involve untruthful strategy from tenants. In [5], the authors proposed an approximately-truthful randomized auction mechanism for emergency DR, which guarantees a 2-approximation of social welfare cost. Nonetheless, both approaches employed combinatorial bidding-based methods, which are NP-hard to arrive at optimal solutions. Furthermore, both approaches are based on reverse auction which can lead to an unexpected number of non-proactive participants. Thus, an advance reward by the operator can avoid a limited tenant participation. In the recent work [6], authors explored emergency DR based on a supply function bidding approach whose drawback is the possible disclosure of tenants' private costs. The work in [13] is also based on Stackelberg-game analysis; however, in this scheme the tenants are required to disclose their cost information, which is undesirable. However, the role of CSP has not been considered in all above works.

II. System Model

A. Overview of the Incentive Mechanism

Let $\mathcal{M}$ be a set of CDCs whose DR service is provided by one CSP, and a set of tenants $\mathcal{N}_i$ is served by a CDC $i \in \mathcal{M}$, where M and N_i denote the corresponding cardinality of the sets. The CSP receives a DR requirement D and decides the compensation price p for each unit from the DR capacity c of all CDCs. Given the price p, each CDC $i \in \mathcal{M}$ decides the energy reduction response q_i (briefly called operator response) and the reward r_i from the reduced energy. Accordingly, with a reward r_i, the tenant $j \in \mathcal{N}_i$ decides an amount of energy reduction $S_j(r_i)$ (a.k.a. tenant supply). However, the operator can also use backup generator to supplement its response deficit when the aggregate tenant supply of the CDCs, $\sum_{j\in\mathcal{N}_i} S_j(r_i)$ is less than the operator response q_i.

B. Tenant Model in Stage III

Consider a rational tenant $j \in \mathcal{N}_i$ of a CDC $i \in \mathcal{M}$. If the reward r_i is known, it can decide to reduce an amount of energy e_j that incurs the cost $V_j(e_j)$ (e.g., such as wear-and-tear, performance degradation [6], [18], etc.) The cost function $V_j(e_j)$ is assumed to be positive, convex, and strictly increasing [6], [17]. Intuitively, the more reduced energy is, the higher cost of tenant is suffered.

Consequently, the rational tenant j will optimally decide the reduced energy e_j^* based on its surplus, $\max_{e_j\ge 0} u_j(e_j) = r_i e_j - V_j(e_j)$. With the standard assumption on $V_j(e_j)$, we applied the cost function as follows

$$V_j(e_j) = \omega_j\, e_j^{\alpha_i}, \quad (1)$$

where ω_j is the *unit cost* and all tenants of CDC i have the same *sensitivity* parameter α_i. The unique tenant supply is obtained as follows

$$S_j(r_i) := e_j^* = V_j'^{-1}(r_i) = \left(\frac{r_i}{\omega_j}\right)^{\frac{1}{\alpha_i-1}} \alpha_i^{\frac{1}{1-\alpha_i}}, \quad (2)$$

which depends on a ratio between the unit reward and cost.

We use the the elasticity of the tenant supply as a measurement for the responsiveness of a firm supply to the price fluctuation in microeconomics [19]. It is related to α_i and we next show the connection as follows:

$$\zeta_j := \frac{r_i S_j'(r_i)}{S_j(r_i)} = \frac{1}{\alpha_i - 1}. \quad (3)$$

We can observe from (3) that price elasticity ζ_j of tenant i only depends on the sensitivity α_i. Moreover, according to (1), higher α_i tenants are more sensitive to energy reduction cost, which indicates its supply is less responsive to a change in the reward, i.e., low ζ_j [19]. We consider $\alpha_i \ge 2$ such that $\zeta_j \le 1$ (see [17] and references therein). Consequently, it enables the diminishing return for tenant supply to prevent infinity the tenant supply when r_i becomes large enough.

Furthermore, given the tenant supply, operator i can make a decision on the response and reward. Nevertheless, due to the tenant's unit cost can be private and/or time-varying [20], the operator cannot obtain the unit costs from tenants. Hence, the uncertainty tenant supply of operator i can be captured in the following tenant supply:

$$S_j(r_i) = \omega_j^{\frac{1}{1-\alpha_i}}\, s_i(r_i),\ \forall j \in \mathcal{N}_i, \quad (4)$$

where $s_i(r_i) := (r_i/\alpha_i)^{\frac{1}{\alpha_i-1}}$. Then, by defining the random variable (R.V.) $\bar{\omega}_i := \omega_j^{\frac{1}{1-\alpha_i}}$, the tenant supply becomes a R.V. as $S_j(r_i) = \bar{\omega}_i s_i(r_i)$. We further assume that $\bar{\omega}_i,\ \forall j \in \mathcal{N}_i$, is i.i.d. with mean μ_i and variance σ_i^2. With an assumption of $\bar{\omega}_i,\ \forall j \in \mathcal{N}_i$, follows i.i.d. with mean μ_i and variance σ_i^2, and aggregated into a new R.V.

$$X_i := \sum\nolimits_{j=1}^{N_i} \bar{\omega}_i. \quad (5)$$

This R.V. has CDF function $F_{X_i}(.)$ and density function $f_{X_i}(.)$ with the non-negative support range $[X_i^l, X_i^u]$. The aggregate tenant supply of CDC i is

$$\sum\nolimits_{j=1}^{N_i} S_j(r_i) = X_i\, s_i(r_i). \quad (6)$$

This supply model is in form of the multiplicative supply model as in [21].

C. CDC Model in Stage II

In this stage, the operator will maximize its expected profit depending on the given CSP compensation price and tenant supply estimation as follows

$$\mathbf{P_1}: \max_{q_i, r_i \ge 0}.\quad \Pi_i^{op}(q_i, r_i; p) := \mathbf{E}\left[p q_i - r_i \sum_{j=1}^{N_i} S_j(r_i) + \gamma\ [D_i]^+\right] \quad (7)$$

$$\text{s.t.}\quad D_i = q_i - \sum\nolimits_{j=1}^{N_i} S_j(r_i), \quad (8)$$

If the aggregate tenant supply is less than the response q_i, the CDC i receives the response deficit D_i with the backup generation (e.g., diesel) unit cost γ. The operator profit $\Pi_i^{op}(q_i, r_i|p)$ is constructed from the revenue $p\, q_i$ (received from the CSP), the incentive cost $r_i \sum_{j=1}^{N_i} S_j(r_i)$, and the backup power cost $\gamma\ [D_i]^+$. We assume all CDCs receive the same γ.

From the problem $\mathbf{P_1}$, the operator will try to reduce tenant's energy consumption first before using the backup generator. This strategy reduces the diesel usage which accounts for high carbon emission [22]. We assume that CDCs cannot predict the impact of its decision to the CSP compensation price. Given p, a profile $(r_i^*(p), q_i^*(p))_{i\in\mathcal{M}}$ is an optimal strategy profile of the operators (i.e. competitive equilibrium) in Stage II by the following condition

$$\Pi_i^{op}(q_i^*, r_i^*; p) \geq \Pi_i^{op}(\bar{q}_i, \bar{r}_i; p), \forall i \in \mathcal{M}, \bar{q}_i, \bar{r}_i \geq 0. \quad (9)$$

For shortage, we ignore the price p in $r_i^*(p), q_i^*(p)$.

D. CSP Model in Stage I

In this stage, the CSP solves the problem $\mathbf{P_2}$ to update the price p and procurement c based on the received responses of CDCs

$$\mathbf{P_2}: \quad \min_{(p,c)\geq 0} \quad \Pi^{lse}(p,c) := p\sum_{i=1}^{M} q_i(p) + \lambda\big(C-c\big)^2$$
$$\text{s.t.} \quad \sum_{i=1}^{M} q_i(p) = c. \quad (10)$$

In $\mathbf{P_2}$, the CSP minimizes the DR cost, including the payment for CDCs to reduce their energy consumption $p\sum_{i=1}^{M} q_i(p)$ and also the penalty $\lambda\,(C-c)^2$. The penalty cost is the multiplication of the penalty rate λ with an amount of the deviation of its procurement c from the target C. We employ this quadratic cost function to reflect the CSP use of alternative energy sources (e.g., real-time market, gas fuel) to bridge the gap $(C-c)$ [17].

III. Operator Decision

From the previous system model, in this section, we show that the operators can solve their problems to achieve the optimal decisions using reward rate for energy procurement. We also prove the existence of a unique optimum at Stage II of the incentive mechanism. We define a new variable $z_i := q_i/s_i(r_i)$ and rewrite the operator profit problem (7) as

$$\begin{aligned}
&\Pi_i^{op}(z_i, r_i;\, p)\\
&= p s_i(r_i) z_i - r_i s_i(r_i)\mathbf{E}\left[X_i\right] - \gamma\, s_i(r_i)\mathbf{E}\left[z_i - X_i\right]^+\\
&= \Upsilon(r_i) - \Theta(z_i, r_i), \quad (11)
\end{aligned}$$

where

$$\Upsilon(r_i) = (p - r_i)s_i(r_i)\mathbf{E}\left[X_i\right] \quad (12)$$
$$\Theta(z_i, r_i) = p s_i(r_i)\big(\mathbf{E}\left[X_i\right] - z_i\big) + \gamma\, s_i(r_i)\mathbf{E}\left[z_i - X_i\right]^+. \quad (13)$$

As the perspective of the classical news-vendor problem, the operator performs two tasks: (a) maximization of its expected risk-less-profit $\Upsilon(r_i)$, and (b) minimization of the expected loss $\Theta(z_i, r_i)$ due to the uncertainty of X_i. Correspondingly, the expected loss contains the trade-off between the opportunity revenue and the expected excess cost (i.e. diesel) as in (13). Clearly, with a high z_i (e.g. $z_i > \mathbf{E}\left[X_i\right]$), the operator receives an opportunity revenue and also incurs a high excess cost. Employing the standard approach to solve the classical news-vendor problem [21], the unique competitive equilibrium solution of problem $\mathbf{P_1}$ for each CDC i can be obtained as follows.

Theorem 1. *There exists a unique optimum $(q_i^*, r_i^*)_{i\in\mathcal{M}}$ at Stage II of the Stackelberg game with a given $p \geq 0$ such that*

$$\begin{aligned}
z_i^* &= \begin{cases} F_{X_i}^{-1}(1) = X_i^u, & \text{if } p \geq \gamma,\\ F_{X_i}^{-1}\Big(\frac{p}{\gamma}\Big), & \text{if } p \leq \gamma, \end{cases}\\
r_i^* &= \begin{cases} \frac{\gamma}{\alpha_i}, & \text{if } p \geq \gamma,\\ \frac{\gamma}{\alpha_i}\cdot\frac{\int_0^{p/\gamma} F_{X_i}^{-1}(w)\,dw}{\mathbf{E}[X_i]}, & \text{if } p \leq \gamma, \end{cases}
\end{aligned} \quad (14)$$

where $F_{X_i}^{-1}$ is the quantile function of F_{X_i}. As a result, $q_i^* = z_i^*\, s_i(r_i^*)$.

Proof: The detail proof is in Appendix A. ■

Some properties can be observed from Theorem 1 as: 1) The operator response problem $\mathbf{P_1}$ is similar to the classing news-vendor problem. Despite the minor difference, the same solution approach can be employed by (a) some algebraic manipulation, and (b) using a sequential optimization approach. 2) We can observe a correlation between z_i^* and X_i in Theorem 1: if the CSP compensation price is higher than diesel cost, the operator chooses $z_i^* = X_i^u$ such that the optimal procurement q_i^* is equal to its maximum tenant supply $X_i^u s_i(r_i)$. 3) If $r_i^* \geq \gamma$, the backup generator produces less cost than the reward. Furthermore, the theorem also presents a stronger result: $r_i^* \leq \gamma/\alpha_i$, i.e., the optimal reward depends on tenant sensitivity α_i. 4) In addition, if $p < \gamma$ the distribution of X_i is needed to derive z_i^*, r_i^*, and q_i^*. Nonetheless, the tenants' cost will be privately evaluated. According to CLT theorem, the optimal solution approximation can be achieved based on μ_i, σ_i and N_i, as follows:

Corollary 1. *If $p \leq \gamma$, according to CLT*

$$z_i^* = \Phi^{-1}(p/\gamma)\sqrt{N_i}\sigma_i + N_i\mu_i, \quad (15)$$
$$r_i^* = \frac{\gamma}{\alpha_i}\cdot\frac{\int_{x_{lo}}^{x_{up}}\big(\sqrt{N_i}\sigma_i\, x + N_i\mu_i\big)\, d\Phi(x)}{N_i\,\mu_i}, \quad (16)$$
$$q_i^* = z_i^*\, s_i(r_i^*), \quad (17)$$

where $x_{lo} = \frac{X_i^{lo} - N_i\mu_i}{\sqrt{N_i}\sigma_i}$, $x_{up} = \Phi^{-1}(p/\gamma)$, and $\Phi^{-1}(\cdot)$ is the quantile of a standard normal distribution.

Proof: Please see Appendix B. ■

IV. CSP Decision

In this section, we present the CSP problem and its solution. According to Theorem 1, the operator response has the following form

$$q_i^*(p) = \begin{cases} q_i^{\max} := F_{X_i}^{-1}(1)\left(\frac{\gamma}{\alpha_i^2}\right)^{\frac{1}{\alpha_i - 1}}, & \text{if } p \geq \gamma;\\ \left(\frac{\gamma}{\alpha_i^2\mathbf{E}[X_i]}\int_0^{p/\gamma} F_{X_i}^{-1}(x)\,dw\right)^{\frac{1}{\alpha_i-1}} F_{X_i}^{-1}\Big(\frac{p}{\gamma}\Big), & \text{if } p \leq \gamma. \end{cases} \quad (18)$$

Then, the CSP's problem $\mathbf{P_2}$ can be rewritten as follows

$$\begin{aligned} \min_{p\geq 0} \quad & \sum\nolimits_{i=1}^{M} p\, q_i^*(p) + \lambda\big(C-c\big)^2 \\ \text{s.t.} \quad & \sum\nolimits_{i=1}^{M} q_i^*(p) = c. \end{aligned} \tag{19}$$

This problem can be solved with local or global solutions by using bisection and branch-and-bound methods, respectively.

V. Numerical Results

A. Operator Decision and Tenant Performance

1) Settings: In this performance analysis, we examine a CDC with $\alpha_i = 2$. Each tenant of this CDC has a random weight ω_i following an uniform distribution in $[0, 1]$ (\$/kWh) and $\bar{\omega}_i$ following an uniform distribution in $(0, 1)$ (\$/kWh). Therefore, X_i follows the Irwin-Hall distribution [23]. The unit cost of backup generator is 0.3 \$/kWh [6].

We generate 100 values of the CSP compensation price from 0 to 0.45 \$/kWh, and the number of tenants $N_i \in [5, 10, 15, 20]$. The performance evaluation is conducted by the comparison between the exact analysis with given X_i distribution from Theorem 1 and the approximation method based on CLT from Corollary 1 when $p < \gamma$.

2) Operator decision: Fig. 1 depicts the operator performance, comprising z_i^*, r_i^*, q_i^*, and the profit $\Pi_i^{op}(z_i^*, r_i^*; p)$ in Figs. 1a, 1b, 1c, and 1d, respectively. In addition, the profit increases by increasing the price p and N_i. Since q^* depends on z_i^* and r_i^*, we see that when $p < \gamma$, due to the specific distribution of X_i that affects both z_i^* and r_i^*, all curves of these three parameters are oddly non-convex. When $p > \gamma$, obviously we have z_i^*, r_i^*, and q^* are constants, while $\Pi_i^{op}(z_i^*, r_i^*; p)$ increases with p.

We note that the CLT approach is a good approximation compared to the exact analysis, and the unique optimal operator profit is attained correctly from our analytic results.

For tenant performance analaysis, we examine via the expected tenant supply $\mathbf{E}[X_i]\, s_i(r_i)$ and the expected tenant surplus with the resultant r_i^* in Fig. 2a, and Fig. 2a, respectively. Obviously, the results show that both performance metrics increase by increasing the price p and N_i.

3) Comparison on exact analysis and CLT-based approximation: By comparing the numerical values depicted in Figs. 1–2, we can conclude that the performance gap between CLT-based approximation and the exact analysis is negligible. Thus, the operator no longer needs to learn X_i distribution.

B. Performance of CSP

1) Settings: Let us consider a CSP with 8 CDCs. The first 4 CDCs host 5, 10, 15 and 20 tenants, correspondingly, and the $\bar{\omega}_i$ following an uniform distribution in $[0, 1]$ (\$/kWh). The remaining 4 CDCs host 20, 40, 80 and 100 tenants, correspondingly, and the tenant weights $\bar{\omega}_i$ are exponentially distributed with mean value 1. This setting allow us to have distinguishable distributions, i.e. the Irwin-Hall and Erlang distributions with different parameters [23], [24], although they have the same number of tenants. Furthermore, the α_i of each CDC varies from 2 to 6, which reflects the heterogeneity of CDC's sensitivity. The Q_{max} is 68 kWh following this setting.

2) Benchmark: The unique optimum of Stage II is compared with the optimal social cost from the following problem:

$$\begin{aligned} \mathbf{P_3}:\ \min_{p,c\geq 0} \quad & \Pi^{soc}(p, c) \\ \text{s.t.} \quad & \sum_{i=1}^{M} q_i(p) = c, \end{aligned} \tag{20}$$

where $\Pi^{soc}(p, c)$ is defined to be

$$\sum_{i=1}^{M}\sum_{j=1}^{N_i} \mathbf{E}\left[V_j(S_j(r_i)) + \gamma\Big[q_i - \sum_{j=1}^{N_i} S_j(r_i)\Big]^+ + \lambda\big(C-c\big)^2 \right], \tag{21}$$

with r_i and q_i is defined in Theorem 1, and $\sum_{j=1}^{N_i} V_j(S_j(r_i)) = \sum_{j=1}^{N_i} \omega_j\, S_j(r_i)^{\alpha_i} = X_i s_i(r_i)^{\alpha_i}$ based on (1) and (2).

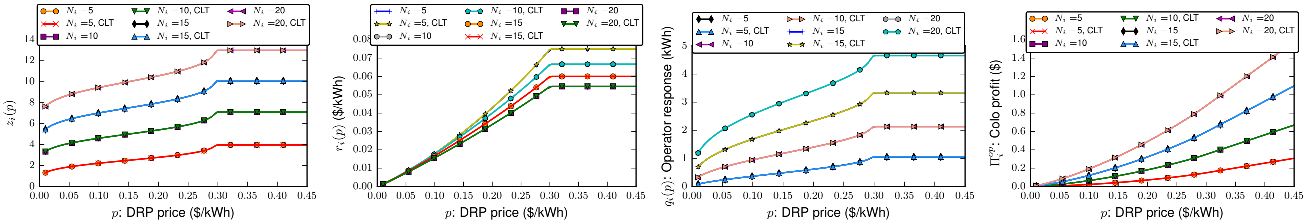
The social cost (SOC) is the aggregate cost from all tenants, operators, and CSP. Since the operator reward to tenants and CSP payment to operators are transferred via the incentive mechanism, they will not affect and be ignored from the social cost. Obviously, the CSP often do not have the tenants cost $V_j(S_j(r_i))$ to solve the SOC problem $\mathbf{P_3}$ [6], [17].

3) Performance metrics: We evaluate two performance metrics: the CSP and social cost with the impact of C and λ. With a generated sequence of independent DR periods, we examine the behaviors of CSP pricing policy according to C and the amount that the CSP can deviate from either its individual cost or social cost according to λ.

We first investigate the impact of C by fixing $\lambda = 0.001$ and evaluating for $C \in [10, 40, 70]$ corresponding to different demand response target levels. Fig. 3a indicates the cost function with respect to price is the convex function, which has a unique optimal solution. As we can observe from the graphs, the minimum CSP cost is greater than the optimal social cost in all cases. Moreover, the difference of CSP and social costs optimal values increases when we increase C.

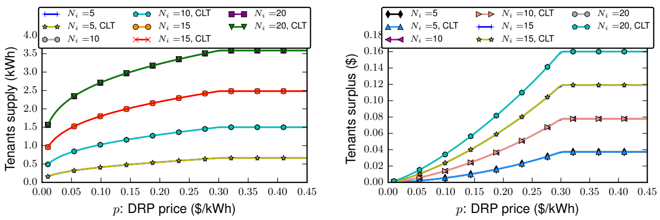
In addition, we investigate the impact of λ by fixing $C = 10$ and evaluating for $\lambda \in [0.001, 0.01, 0.1]$ corresponding to different penalty rate levels. Fig. 3b depicts the trajectories of CSP and social cost with respect to prices and the difference of CSP and social costs optimal values increases when we increase λ.

Finally, we compare the optimal prices and costs of individual and social objectives of the CSP in Fig. 4 and Fig. 5. We fix $\lambda = 0.001$ and increase C from 10 to 70, then both optimal prices and optimal costs increase in Fig. 4a, and Fig. 5a, respectively. Moreover, the difference of the minimum CSP's individual and social prices increases and either the difference of the CSP's individual and social cost. As we can observe from Fig. 5a, in order to balance the deviation cost $\lambda\,(C-c)^2$ with CSP cost $\sum_{i=1}^{M} p\, q_i^*(p)$, CSP provides lower prices when the social price is increased. We next fix $C = 10$ and increase values of λ from 0.001 to 0.1 in Fig. 4b, and Fig. 5b, respectively. We can observe that the CSP minimum



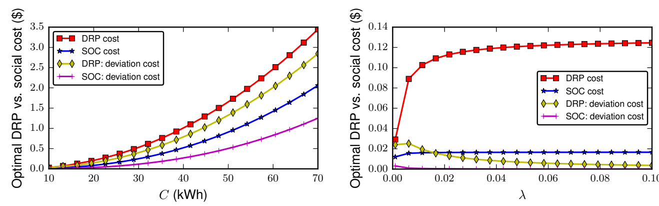
(a) z_i (b) r_i (c) q_i (d) Π_i

Fig. 1: CDC operator parameters with different N_i



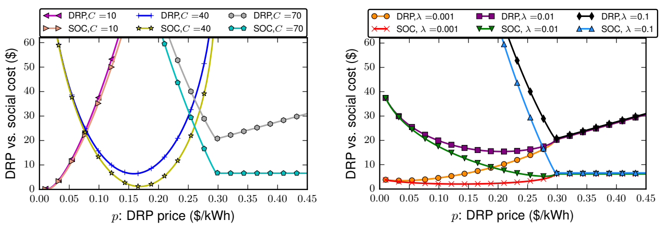
(a) Tenant response. (b) Tenant surplus.

Fig. 2: Tenant performance.



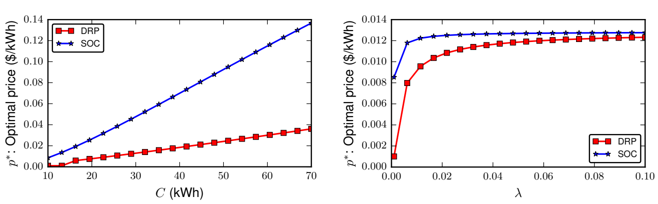
(a) Increasing C. (b) Increasing λ.

Fig. 5: Comparison of optimal CSP cost with social cost.



(a) Fix λ, vary C. (b) Fix C, vary λ.

Fig. 3: Comparison of CSP and social costs with different prices.



(a) Increasing C. (b) Increasing λ.

Fig. 4: Comparison of optimal CSP compensation price with social price.

price the social price both rapidly increase up till a constant and the similar trend for the optimal CSP and social costs. Due to CSP tries to produce low the price for its own benefit, then a sufficiently high λ makes the deviation cost becomes dominant.

VI. Conclusions

Our work designs an incentive mechanism for CDC economic DR by considering the role of CSP. The proposed incentive pricing mechanism encourages CDC to curtail its energy consumption. The designed mechanism comprises three-stage decisions, i.e. the CSP determine its compensation price for CDCs, then CDCs compute the reward for tenants and an amount of the energy reduction. The strategic interactions between CSP, CDCs and tenants are formulated as a three-stage Stackelberg game. We also showed that a Stackelberg equilibrium exists where CSP, CDCs and tenants make optimal decisions. In addition, we present a near-optimal approximate approach for Stage II. We validate the approximation methods by extensive numerical study. We provide extensive numerical study and presented the comparison of the optimal CSP compensation price (cost) with the social price (cost).

Appendix A
Proof of Theorem 1

We use the iterative optimization approach as in [21]. By fixing r_i, we can obtain the z_i^*. We derive the first and second partial derivatives and optimal z_i^* as follows

$$\frac{\partial \Pi_i^{op}}{\partial z_i} = s_i(r_i)\big(p - \gamma\, F_{X_i}(z)\big), \tag{22}$$

$$\frac{\partial^2 \Pi_i^{op}}{\partial z_i^2} = -\gamma\, s_i(r_i)\, f_i(z) < 0, \tag{23}$$

$$z_i^* = \begin{cases} F_{X_i}^{-1}\left(\frac{p}{\gamma}\right), & \text{if } p \le \gamma; \\ F_{X_i}^{-1}(1) = X_i^u, & \text{if } p \ge \gamma. \end{cases} \tag{24}$$

Hence, we have

$$\begin{aligned} &\Theta(z_i^*, r_i) \\ &= p s_i(r_i)\big(\mathbf{E}\,[X_i] - z_i^*\big) + \gamma s_i(r_i)\mathbf{E}\,[z_i^* - X_i]^+ \\ &= p s_i(r_i)\big(\mathbf{E}\,[X_i] - z_i^*\big) + \gamma s_i(r_i) \int_0^{z_i^*} (z_i^* - u)\, f_i(u)\, du \end{aligned}$$

$$= ps_i(r_i)\big(\mathbf{E}[X_i] - z_i^*\big) + \gamma s_i(r_i) \int_0^{p/\gamma} \big(z_i^* - F_{X_i}^{-1}(w)\big)\, dw$$

$$= ps_i(r_i)\mathbf{E}[X_i] - \gamma s_i(r_i) \int_0^{p/\gamma} F_{X_i}^{-1}(w)\, dw. \quad (25)$$

In the third equality of (25), we replace the variable $w = F_{X_i}(u)$ then $u = F_{X_i}^{-1}(w)$.

Therefore, we have

$$\Pi_i^{op}(z_i^*, r_i) = \Upsilon(r_i) + \Theta(z_i^*, r_i) \quad (26)$$
$$= -r_i s_i(r_i)\mathbf{E}[X_i] + \gamma s_i(r_i) \int_0^{p/\gamma} F_{X_i}^{-1}(w)\, dw.$$

After receiving z_i^*, we next derive r_i^* from the first-order condition

$$\frac{\partial \Pi_i^{op}(z_i^*, r_i)}{\partial r_i} \quad (27)$$
$$= -\big(r_i s'(r_i) + s_i(r_i)\big)\mathbf{E}[X_i] + \gamma s'(r_i) \int_0^{p/\gamma} F_{X_i}^{-1}(w)\, dw$$
$$= s'(r_i)\Big[-r_i \mathbf{E}[X_i] + \gamma \int_0^{p/\gamma} F_{X_i}^{-1}(w)\, dw\Big] - s_i(r_i)\mathbf{E}[X_i]$$
$$= \frac{1}{\alpha_i - 1}\Big[-\mathbf{E}[X_i] + \frac{\gamma}{r_i} \int_0^{p/\gamma} F_{X_i}^{-1}(w)\, dw\Big] - \mathbf{E}[X_i] = 0.$$

The last equality can be derived from (3) as follows

$$\frac{r_i s_j'(r_i)}{s_j(r_i)} = \frac{r_i S_j'(r_i)}{S_j(r_i)} = \zeta_j = \frac{1}{\alpha_i - 1}. \quad (28)$$

Therefore, we have

$$r_i^* = \frac{\gamma}{\alpha_i} \cdot \frac{\int_0^{p/\gamma} F_{X_i}^{-1}(w)\, dw}{\mathbf{E}[X_i]}. \quad (29)$$

APPENDIX B
PROOF OF COROLLARY 1

As in Theorem 1, if $p \le \gamma$ then $z_i^* = F_{X_i}^{-1}\left(\frac{p}{\gamma}\right)$

$$\Longrightarrow\ p/\gamma = F_{X_i}(z_i^*) = \Phi\left(\frac{z_i^* - N_i\mu_i}{\sigma_i\sqrt{N_i}}\right), \quad (30)$$

which implies (15). The equality (30) can be derived by the CLT, where X_i is the sum of N_i i.i.d. R.V's.

Consider r^* in (14), by changing variable $\omega = F_{X_i}(u)$ such that $u = F_{X_i}^{-1}(w)$, we have

$$r_i^* = \frac{\gamma}{\alpha_i} \cdot \frac{\int_0^{p/\gamma} F_{X_i}^{-1}(w)\, dw}{\mathbf{E}[X_i]} = \frac{\gamma \int_{X_i^l}^{F_{X_i}^{-1}\left(\frac{p}{\gamma}\right)} u\, dF_{X_i}(u)}{\alpha_i \mathbf{E}[X_i]}. \quad (31)$$

We again apply the CLT as

$$x = \frac{u - N_i\mu_i}{\sigma_i\sqrt{N_i}} \implies F_{X_i}(u) = \Phi\left(\frac{u - N_i\mu_i}{\sigma_i\sqrt{N_i}}\right) = \Phi(x). \quad (32)$$

Substituting (32) to (31), and using $\mathbf{E}[X_i] = N_i\mu_i$, we obtain (16).

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