

Pricing Mechanism for Virtualized Heterogeneous Resources in Wireless Network Virtualization

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Abstract—Virtualized resource in the wireless network enables to share the physical resource efficiently to increase the resource utilization. In the wireless network virtualization, an infrastructure provider (InP) and mobile virtual network operator (MVNO) can design a pricing scheme for the network resource to maximize their revenue and network resource sharing. Previous works have been proposed pricing mechanisms for virtualized resource sharing and allocation as an optimal solution. However, previous works mainly focus on a single type of virtualized resource. Thus, in this paper, we propose a pricing mechanism for multiple types of virtualized resources that include communication and caching resources in the wireless network. To do this, we formulate an optimization problem by considering social welfare among the MVNOs and show that this problem is NP-hard. Therefore, to solve the formulated problem, we propose an auction-based solution approach. The auction allows placing multiple MVNOs together in the same competitive market of virtualized heterogeneous resources. In our auction approach, first, InP decides and announces the reservation price to the mobile virtual network operator which covers capital expenditures (CAPEX) and Operating Expenditure (OPEX). Second, depending on resource demand to satisfy users' requests, mobile virtual network operators determine and submit the bidding value to InP. Finally, InP evaluates bidding values from MVNOs to select the winning MVNOs and decide the optimal price for virtualized heterogeneous resources. The performance of the proposed algorithm shows that our approach can guarantee social welfare maximization.

Index Terms—resource allocation, auction, pricing, virtualized resource, wireless network virtualization

I. INTRODUCTION

To enable heterogeneous stringent requirements posed by the user in 5G wireless network, a virtualized wireless network has emerged as one of the most promising solution [1]–[3]. Enabling wireless network virtualization, the efficiency of resource sharing and utilization rate can be guaranteed by the allocated virtualized resource which is divided from the physical resource. In the modern wireless system, resources

such as communication, caching, and computing are shared among users based on their demands [4], [5]. Moreover, specifically, the popularity of compute-intensive applications in wireless networks has seen a dramatic surge of computing resources along with traditional resources in modern wireless networks. Thus, to allocate the resources efficiently, a number of solutions have been presented [6], [7]. In [8], the author has proposed the Generalized Kelly Mechanism-based resource allocation method. In addition, they have solved the two-level allocation problem in network slicing with inter and intra slice isolation. In [9], The hierarchical matching game was proposed for resource allocation. Here, matching based service selection occurs for user association between mobile virtual network operator (MVNO) and user so that provide the slice as the resource by purchasing from infrastructure provider (InP).

Furthermore, pricing based resource allocation is also considered as a popular method [10]. The purpose of the pricing based resource allocation is that seller who has their own resource share the resource with others based on the competition of the resource price which is bidding price from the buyers. Based on the bidding price from the buyer, the winner is selected and the optimal price is decided by considering the revenue of the buyer and seller. For this purpose, the auction-based solution was proposed as an economic and business management approach. In [11], auction-based efficient allocate method was proposed for the infinitesimal divisible resource. In addition, the proposed mechanism is based on Vickrey-Clarke-Groves (VCG) auction with a two-dimensional bid which specifies a per-unit price and maximum of the demand. Also, In [12], the VCG auction was proposed for resource block allocation between InP and MVNOs in the Long-Term Evolution (LTE) environment. Furthermore, In [13], the authors proposed a combinatorial double auction as a solution for resource allocation. In the auction model, the broker (i.e., auctioneer) was considered who manage the auction.

In this paper, we propose a pricing mechanism for bandwidth and cache resources allocation in the wireless network virtualization. In our approach, the differentiation point of our

This work was supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MSIT) (NRF-2017R1A2A2A05000995). *Dr. CS Hong is the corresponding author.

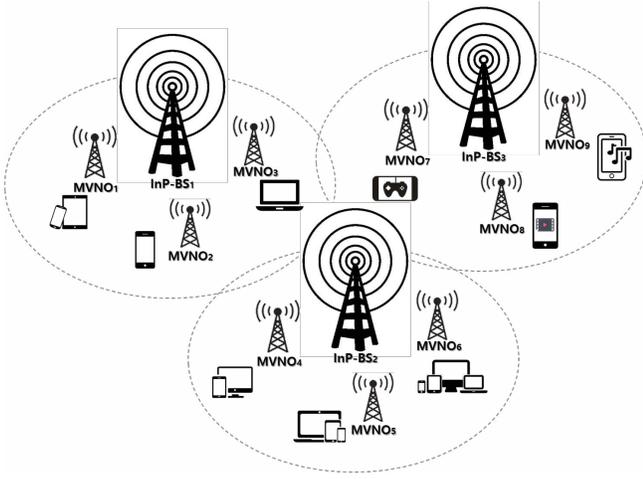


Fig. 1. System Model

approach with the aforementioned work is that we consider the heterogeneous resources (i.e., communication, cache resource). The auction procedure of our approach is conducted without an external broker because the charges that needed to be paid to the broker (i.e., auctioneer) increase the resource price. Furthermore, the complexity of the auction process, such as collecting bid, checking the available resource, finding the winners can be reduced by our approach. Hence, the pricing mechanism without any broker is considered in this paper.

The rest of this paper is organized as follows. In Section II, we describe the system model of this paper. Section III presents the formulation of the problem and describes our pricing mechanism for virtualized heterogeneous resources in detail. In Section IV, we present a performance evaluation for the proposed solution in terms of optimal price and social welfare. Finally, we conclude and present the future work for this paper in Section V.

II. SYSTEM MODEL

We consider that we have one infrastructure provider (InP) that covers a certain geographic area denoted A using N_{BS} based stations (BSs), where N_{BS} represents the number of BSs. Therefore, the number of BSs needed to cover a geographic area can be expressed as follows:

$$N_{BS} = \frac{A}{\pi L^2} \quad (1)$$

where L is the radius of the area covered by one BS. We assume L is the same value for all the BSs of InP. Furthermore, we consider that the resources of each BS are virtualized for being allocated to multiple MVNO, where S is the number of slices per each BS. We consider \mathcal{M} as a set of m MVNOs, where each MVNO can be assigned one slice with its associated resources, i.e., communication (bandwidth) and caching resources.

A. Operational Expenditure of InP

In this subsection, we analyze the Operational Expenditure (OPEX) of InP. As described and proved in [14], we assume

that the power consumption is dominant in OPEX of InP. In the power consumption of BS, we consider the following categories of the power: transceiver power P_t , rectifier power P_r , digital signal processor power P_d , air cooling power P_c , amplifier power P_a , and microwave transmission power P_m . Furthermore, it is approved in [14] that the P_c and P_m increase as the number of slice S increases. Therefore, as described in [14], P_c and P_m can be expressed as follow:

$$P_c = P_c[1 + \sigma(S_n - 1)] \quad (2)$$

$$P_m = P_m[1 + \sigma(S_n - 1)] \quad (3)$$

where σ is the weight parameter. The total power consumption P is given by:

$$P = N_{BS} * S * N_a (P_t + P_r + P_d + P_a) + P_c + P_m \quad (4)$$

where N_a is the number of antenna per BS. We assume that the InP can serve a certain number of users N_{UE} of all MVNOs, where we use N_{UE} to denote the number of users, where each user of MVNO can have a maximum bandwidth of capacity r_{UE} Mbps. Therefore, the total bandwidth R needed to serve N_{UE} is given by:

$$R = r_{UE} N_{UE} \quad (5)$$

Therefore, the power consumed per bit $P_{bit} = \frac{P}{R}$. Furthermore, based on bandwidth demand r_m for each MVNO m , we can calculate the OPEX of InP for communication bandwidth B_{OP} as follows:

$$B_{OP} = e_p \sum_{m=1}^M P_{bit} r_m \quad (6)$$

where e_p is the unit price of the power.

In addition to communication bandwidth, we consider that InP has caching resources at BSs, where caching resources are also virtualized. OPEX of InP for caching resource C_{OP} can be calculated as follows:

$$C_{OP} = N_{BS} S c_p c_s \quad (7)$$

where c_s is cache storage needed for each slice and c_p is the unit price of cache storage. Therefore, the total OPEX of InP can be expressed as follows:

$$I_{OP} = C_{OP} + B_{OP} \quad (8)$$

B. Revenue of InP

Let us consider the InP revenue is the total amount of income generated by selling different resources to MVNOs. Based on the prediction of demands for caching and bandwidth resources, we assume that each MVNO knows its need for both caching and bandwidth resources. Therefore, to increase the revenue, the InP sells caching and bandwidth resources to MVNOs.

The system model shown in Fig. 1, we consider that InP has a total virtualized cache resource $C = N_{BS} S c_s$ and bandwidth R which are virtualized for being allocated to multiple MVNOs on payments. We consider each MVNO $m \in \mathcal{M}$ to be strategic and selfish, i.e., there is no cooperation

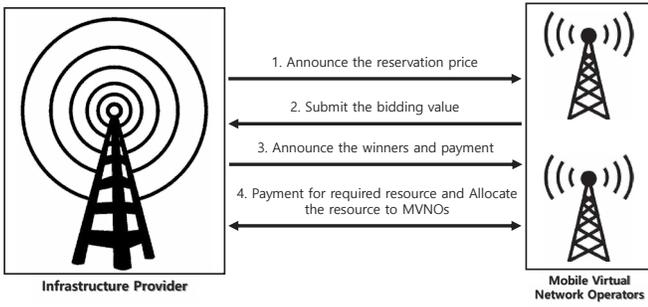


Fig. 2. Workflow of Auction Model

between MVNOs. Furthermore, we consider that we need an algorithm that allows the InP to efficiently sell different virtualized resources to maximize its profits. Therefore, we consider maximizing the profits of all MVNOs and InP as social welfare maximization, and this motivates us to use Vickrey-Clarke-Groves (VCG) in our system model. The VCG mechanism [15] allows social welfare maximization. In VCG mechanism, we use an auction, where we consider InP as a seller and MVNOs as the buyers of different virtualized resources. Here, we consider communication bandwidth R and caching resource C . Furthermore, to ensure that the InP sells its resources to MVNOs on the prices that help the InP to cover its OPEX and guarantee its business continuity, we introduced the following reservation prices for caching C_{resv} and communication resource B_{resv} .

$$C_{resv} = \frac{C_{OP}}{M^s} \quad (9)$$

$$B_{resv} = \frac{B_{OP}}{M^s} \quad (10)$$

where M^s is the maximum number of MVNOs that can be served by the InP.

Based on (9) and (10), we introduce an auction between InPs and MVNOs that maximizes social welfare, where the InP runs the auction to determine the winning MVNOs for resources selling. The workflow of our auction model described in Fig. 2 for efficiently purchasing virtualized resources is described as follows:

- In the first step, the InP announces its reservation prices C_{resv} per unit of cache resource C and B_{resv} per unit of bandwidth resources R .
- In step two, each MVNO $m \in \mathcal{M}$ who needs resources submit a bid $(b_m(c), b_m(r), c_m, r_m)$ to InP for purchasing resources. In the bid, $b_m(c)$ is the bidding value per unit of cache resource and c_m is total needed cache resource for each MVNO $m \in \mathcal{M}$. In addition, $b_m(r)$ is the bidding value per unit of bandwidth and r_m is total needed bandwidth for each MVNO $m \in \mathcal{M}$.
- In step three, the InP collects the bidding value from MVNOs and performs winner determination and social-optimal prices determination, where C_{pm}^* is the optimal price for caching resource and B_{pm}^* is the optimal price

for bandwidth that each winning MVNO $m \in \mathcal{W}$ has to pay for the resources. We use \mathcal{W} to denote a set of winners, i.e., winning MVNOs, where \mathcal{W} is a subset of \mathcal{M} , i.e., \mathcal{W} is a set of MVNOs that win the auction. After winner and price determination, the InP announces the winners and the resources allocated to them.

- Finally, in the last step, each winning MVNO $m \in \mathcal{W}$ pays C_{pm}^* and B_{pm}^* to InP for resources, where $C_{pm}^* \geq C_{resv}$ and $B_{pm}^* \geq B_{resv}$. Then, the InP allocates the resources to each winning MVNO $m \in \mathcal{W}$

III. PROBLEM FORMULATION AND SOLUTION APPROACH

In this section, we formulate our problem and describe the pricing mechanism as a solution in detail.

A. Problem Formulation

In our problem formulation, we define x_m as a decision variable that indicates whether the InP can serve the MVNO m . We consider $x_m = 1$ when the InP can satisfy the demand of each winning MVNO $m \in \mathcal{M}$, otherwise $x_m = 0$. If MVNO m wins the auction, it has to pay to C_{pm}^* for caching resource and B_{pm}^* for bandwidth resource to the InP. Therefore, the utility of InP gained from selling different resources to MVNOs is given by:

$$U_{InP} = \sum_{m=1}^M x_m (B_{pm}^* + C_{pm}^*) \quad (11)$$

Therefore, for determining the optimal price $O_m^* = B_{pm}^* + C_{pm}^*$ that each MVNO m have to pay the InP, we formulate social welfare maximization as an optimization problem described below:

$$\mathbb{P} : \max_{\mathbf{x}} \sum_{m=1}^M x_m (b_m(c) + b_m(r)) \quad (12)$$

$$s.t. \quad \sum_{m=1}^M x_m r_m \leq R \quad (12a)$$

$$\sum_{m=1}^M x_m c_m \leq C \quad (12b)$$

$$\sum_{m=1}^M x_m b_m(c) \geq C_{OP} \quad (12c)$$

$$\sum_{m=1}^M x_m b_m(r) \geq B_{OP} \quad (12d)$$

$$x_m \in \{0, 1\} \quad (12e)$$

In the above-formulated problem in (12), the constants in (12a) and (12b) ensure that the virtualized communication and cache resource allocated to MVNOs should be less than or equal to the total available communication and caching resources of InP. Furthermore, the constraints in (12c) and (12d) guarantees that the InP selling for communication and caching resources covers OPEX of the InP.

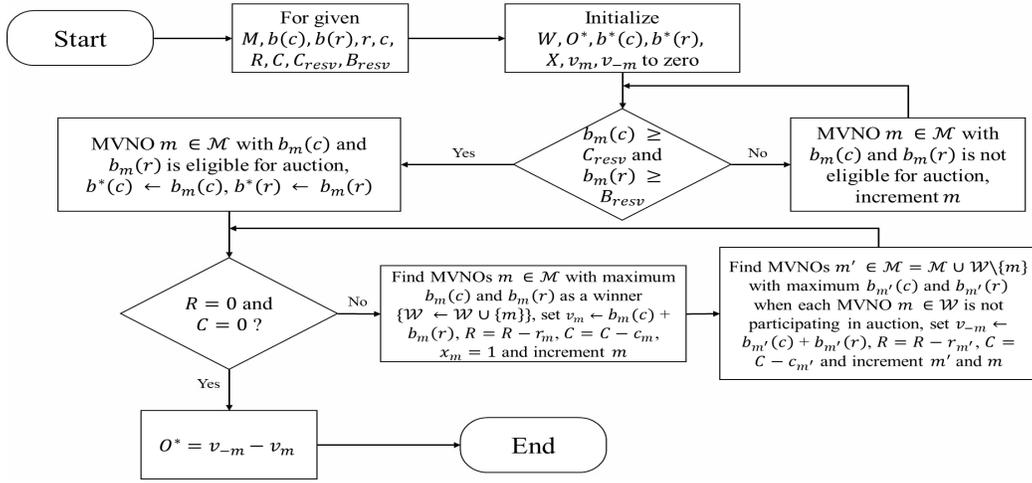


Fig. 3. Algorithm for Selling Different Resources to MVNOs

B. Solution Approach

We consider that each MVNO $m \in \mathcal{M}$ has the following true valuation $V_m = v_m(r) + v_m(c)$, where $v_m(r)$ is the true valuation for communication resource and $v_m(c)$ is the true valuation for caching resource. Suppose each MVNO m submits bid $(b_m(c), b_m(r), c_m, r_m)$ for communication and caching resources, if MVNO m wins the auction, it pays $O_m^* = B_{pm}^* + C_{pm}^*$ to InP, and 0 otherwise. In other words, the MVNOs who loss in the auction pays nothing. Therefore, the utility of each MVNO $m \in \mathcal{M}$ can be expressed as follows:

$$U_m = \begin{cases} V_m - x_m O_m^*, & \text{if MVNO } m \in \mathcal{W}, \\ 0, & \text{otherwise,} \end{cases} \quad (13)$$

where \mathcal{W} is a set of winners.

In our auction, the individual rationality is achieved if and only if for every bidder MVNO m , $U_m \geq 0$. Therefore, to ensure individual rationality and truthful bidding, we use the VCG mechanism [15]. In VCG, each bidder pays the harm that causes to other bidders. Therefore, the welfare of other players than MVNO $m \in \mathcal{W}$ from the chosen outcome when MVNO m is not participating in auction is given by:

$$v_{-m} = \sum_{m'=1}^{M \setminus \{m\}} (b_{m'}(c) + b_{m'}(r)), \quad (14)$$

$$m' \neq m, m' \in \mathcal{M} \setminus \{m\}, m \in \mathcal{W}$$

On the other hand, the welfare for other bidders than MVNO m from the chosen outcome when MVNO $m \in \mathcal{W}$ participate in auction is expressed as follows:

$$v_m = \sum_{m'=1}^M (b_{m'}(c) + b_{m'}(r)), \quad (15)$$

$$m' \neq m, m' \in \mathcal{M}, m \in \mathcal{W}$$

When MVNO m participates in the auction, the optimal price that each MVNO has to pay for a different virtualized resource is given by:

$$O_m^* = v_{-m} - v_m \quad (16)$$

Therefore, Utility of InP is expressed as follows:

$$U_{InP} = \sum_{m=1}^M x_m (B_{pm}^* + C_{pm}^*) = \sum_{m=1}^M x_m O_m^* \quad (17)$$

In order to reduce the computational complexity of the formulate problem in (12), which is an integer linear programming (ILP) problem and NP-hard, we propose a VCG-based algorithm as shown in Fig. 3 for selling different resources to MVNOs. In the proposed algorithm as shown in Fig. 3, the set of bidders \mathcal{M} , reservation prices for resources C_{resv} and B_{resv} , the vector of bidding values for caching resource $b(c)$ and for communication resource $b(r)$, the vector of needed cache resource c , vector of needed communication resource r , the total cache resource C , and the total communication resource R are given as inputs. We initialize the set of winners \mathcal{W} , the vector of payment O^* , the vector $b^*(c)$, $b^*(r)$ of all bidders that bids $b_m(c) \geq C_{resv}$ and $b_m(r) \geq B_{resv}$, and the vector X of decision variable x_m . When $b_m(c) \geq C_{resv}$ and $b_m(r) \geq B_{resv}$, the algorithm starts the iteration for finding winning MVNOs, i.e., the MVNOs that provide maximum bidding values until resources $R = 0$ and $C = 0$ or there is no remaining MVNO that needs resources. Based on VCG, we analyze the impact of each bidder that may cause other bidders by participating in auction and come-up with the optimal prices $O^* = v_{-m} - v_m$ that each bidder has to pay to InP for its caching and communication resources.

IV. PERFORMANCE EVALUATION

In the performance evaluation, we use numerical analysis, where the julia language [16] is used. In the auction, we use $M = 10$ MVNOs, the demands for virtualized cache resources are in the range from $c_m = 100$ Gigabytes(GB) to $c_m = 500$

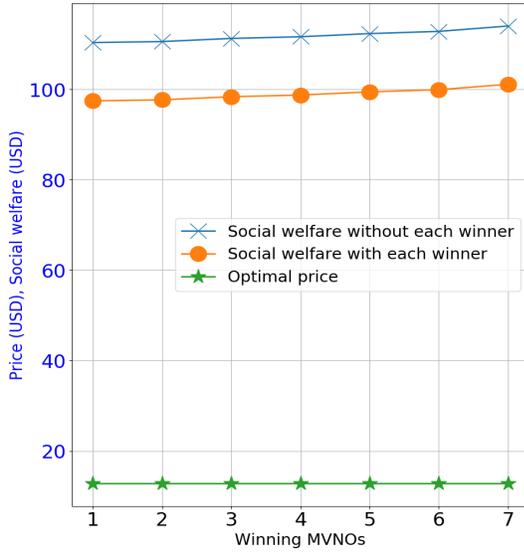


Fig. 4. Social Welfare and Optimal Price per unit of Cache and Bandwidth Resources

GB, and the demands for required data rate are in range from $r_m = 40$ Mbps to $r_m = 100$ Mbps. In addition, we consider the reservation price for cache resources to be $C_{resv} = \$3.31$ per 1 GB [17], while the reservation price for bandwidth to be $B_{resv} = \$8$ per 1 Mbps [18]. Furthermore, we consider the bidding values of MVNOs for cache resources to be in range from $b_m(c) = \$3.31$ to $b_m(c) = \$8.31$, while the total virtualize cache resource for InP is $C = 2000$ GB. In addition, the bidding value of MVNOs for bandwidth are in range from $b_m(r) = \$8$ to $b_m(r) = \$10$ [19] while the maximum data rate for InP is $R = 3500$ Mbps. we assume that based on prediction of needed cache size and data rate to satisfy the demands of MVNOs, we consider that the auction is performed at the end of day, where the MVNOs buy virtualized cache and bandwidth resources to satisfy the demands of MVNOs for the next day.

The simulation results in Fig. 4 show optimal price $O_m^* = 12.889$ for each winning MVNOs and social welfares v_{-m} (without each winning MVNO $m \in \mathcal{W}$) and v_m (with each winning MVNO $m \in \mathcal{W}$) per unit of cache storage (1GB) and bandwidth (1Mbps). Although we use 10 MVNOs, based on the available size of the virtualized cache and bandwidth resources, only 7 MVNOs won the auction. Also, our VCG-based auction guarantees social welfare maximization, where social welfare is increasing for all MVNOs.

Fig. 5 shows the result of cache and bandwidth resource allocation to each winning MVNOs. The increase of cache resources goes with the backhaul bandwidth saving, which increases the profits of MVNOs. If MVNO uses the cache resource from Data Center, MVNO has to pay for the usage of the backhaul bandwidth. Thus, payment for backhaul bandwidth is saving if MVNO buys the cache resource from InP.

The revenue of InP comes from the total amount of income generated by selling caching and bandwidth resources to MVNOs. In Fig. 6, we demonstrate the revenue of InP, which

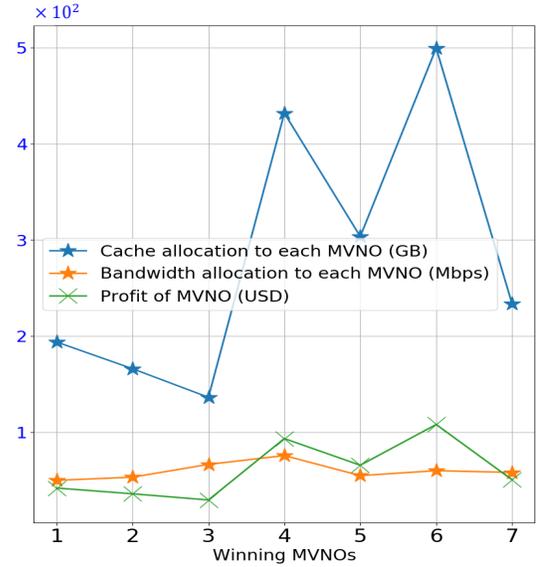


Fig. 5. Result of Resource Allocation and Profit of MVNO

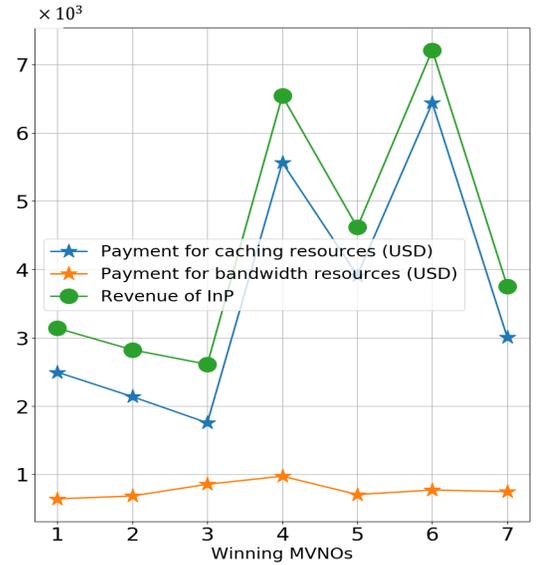


Fig. 6. Revenue of InP and MVNOs' Payments for Caching and Bandwidth Resources

is the sum of the payments from MVNOs for caching and bandwidth resources. By comparing Fig. 5 and Fig. 6, we can see that the graph of payments for bandwidth and caching resources has the same slope of the graph of bandwidth and caching resource allocation. In other words, each MVNO pays its resources based on its demands and optimal price.

V. CONCLUSION

In this paper, we have proposed the pricing mechanism based on VCG auction for resource allocation between InP and MVNOs, where we have considered communication (bandwidth) and cache resources. In our solution approach, the auction procedure is conducted without an external broker to reduce the complexity of the auction process, such as collecting bid, checking the available resource, finding the winners.

Simulation results show that our approach can guarantee social welfare maximization and can be applied to allocate the heterogeneous resources.

To decide the demand for the resource in the MVNO side, there are two possibilities. The first is the assumption that the demands for resource purchasing are already known for MVNO. The second is that demands will be known based on the prediction algorithm (e.g., deep learning). Thus, for future work, we will consider the prediction algorithm based on deep learning to forecast the required resource and apply it to the proposed mechanism. Furthermore, to evaluate the efficiency of our approach, we will conduct a comparison with the other methods, such as a random, proportional approach.

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