Matching-based Distributed Resource Allocation in Cognitive Femtocell Networks

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***Abstract*—In this paper, a novel framework is proposed for joint subchannel assignment and power allocation in the uplink of cognitive femtocell network (CFN). In the studied model, femtocell base stations (FBSs) are deployed to serve a set of femtocell user equipments (FUEs) by reusing subchannels in a macrocell network. The problem of optimal allocation of subchannels and transmit power is formulated as an optimization problem in which the goal is to maximize the overall uplink throughput while guaranteeing minimum rate requirement of the served FUEs and macrocell base station (MBS) protection. To solve this problem, a novel framework based on matching theory is proposed to model and analyze the competitive behaviors among the FUEs and FBSs. Using this framework, distributed algorithms are implemented to enable the CFN to make decisions on subchannel allocation and power control. The developed algorithms are then shown to converge to stable matchings. Simulation results show that the proposed approach yields a notable performance improvement, in terms of the overall network throughput and outage probability while requiring only a small number of iterations for convergence.**

I. Introduction

The use of small cell networks based on the pervasive deployment of low-power, low-cost femtocell base stations (FBSs) is seen as a promising technique to improve the capacity, and enhance the coverage for indoor and cell edge users in next-generation wireless cellular networks [1]. In order to utilize the limited licensed spectrum efficiently, FBSs must reuse the same orthogonal frequency multiple access (OFDMA) radio resources as the macrocell network [2]. Frequency reuse in two-tier networks composed of FBSs coexisting with macrocell base stations (MBSs) can lead to a problem of co-channel interference thus requiring a smart adaptation of scheduling algorithms to mitigate the co-channel interferences among users using the same channel [2]. Cognitive radio (CR) can be a promising technology to realize such flexible interference management. A femtocell network that reuses subchannels using CR technology is commonly known as cognitive femtocell network (CFN) [3].

A CFN can operate successfully and cost-efficiently by using one of two cognitive radio network (CRN) spectrum sharing approaches: overlay and underlay spectrum access approaches [3]. In the overlay spectrum access, secondary users (SUs) can use the licensed spectrum when it is vacant and not occupied by the licensed primary users (PUs). In underlay spectrum access, SUs are allowed to simultaneously operate in frequency bands where PUs are active, while the overall interference from SUs' occupancy on the same frequency band to the PU receiver should be kept below a given threshold. In this approach, entities are assumed to have knowledge of the interference caused by transmitters in the primary network [4]. Here, our focus is on CFNs which are based on a spectrum underlay approach such as in [4]–[6].

To reap the benefits of CFN deployment, some technical challenges such as interference management, efficient spectrum usage, and cell association must be addressed [2], [3], [7] and [8]. Several recent studies have studied resource allocation in uplink OFDMA cognitive femtocell network [4]–[6], [9]–[14]. *Power control* in the uplink of two-tier networks has been studied in [4], [5] and [9]. The works in [5] and [9] only considered access and power control in the single-channel operation to maximize total network throughput. Distributed power control for spectrum-sharing femtocell networks using a Stackelberg game is proposed in [4]. However, these works in [4], [5] and [9] assume that subchannel allocation is predetermined and not optimized. *Subchannel assignment* have been studied for uplink OFDMA-based femtocell network in [12]–[14]. However, such existing works do not consider power control in proposed resource allocation algorithms in which transmit power of users are fixed and not optimized. Moreover, the problem of *joint* subchannel assignment and power allocation in uplink OFDMA-based femtocell networks was investigated in [6], [10] and [11]. A distributed power control and centralized matching algorithms for subchannel allocation is proposed in [10] that leads to fair resource allocation for uplink OFDMA femtocell networks. Additionally, the authors in [6] investigated the joint uplink subchannel and power allocation problem in cognitive small cells using cooperative game theory. However, none of works in [6], [10] and [11] studies resource allocation in which both users and base stations participate in the subchannel association problem.

The main contribution of this paper is to introduce a novel framework for joint subchannel assignment and power allocation in the uplink of CFNs. In particular, our contributions can be summarized as follows:

- We investigate the joint *subchannel* and *power allocation* problem in the uplink of an underlay CFN, and we show that, using centralized optimization, such a problem is NP-hard.

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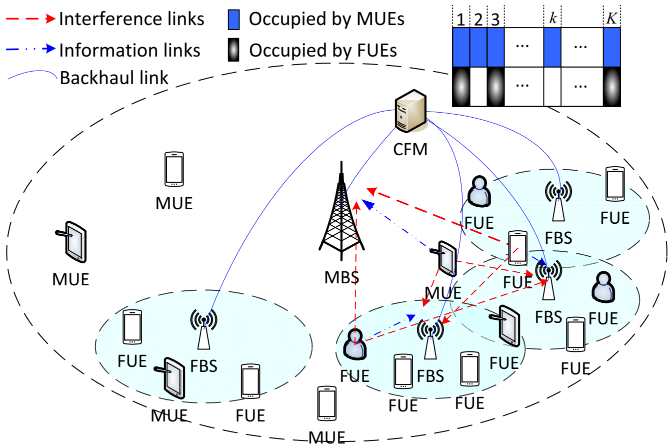


Figure 1: System architecture of a cognitive femtocell system.

- To overcome this complexity, we formulate the problem as a matching game between FUEs and FBSs which enables us to properly capture competitive behaviors of FUEs and FBSs while seeking to maximize the overall uplink throughput.
- To solve this game, we design an algorithm that jointly allocate subchannels to FUEs and allocates transmit power level to FUEs. Then, we prove that the proposed algorithms converge to the group stable matchings.
- Simulations results show that the proposed algorithms converge to stable outcomes for whole system after a small number of iterations. These results also show that the proposed approach yields a performance improvement, in terms of the overall network throughput.

The rest of this paper is organized as follows: Section II explains the system model and problem formulation. The optimization problem is solved based on dynamic matching game in Section III. In Section IV, we study convergence and stability of the proposed algorithms. Section V provides simulation results. Finally, conclusions are drawn in Section VI.

II. System model and problem formulation

A. System model

Consider the *uplink* of an OFDMA cognitive femtocell network composed of a set $\mathcal{M} = \{1, 2, ..., M\}$ of FBSs that operate inside the coverage of a macrocell network and serve a set $\mathcal{N}$ of FUEs as shown in Fig. 1. Each FBS $m \in \mathcal{M}$ is associated to a set $\mathcal{N}_m = \{1, 2, ..., N_m\}$ of FUEs, $\sum_{m\in\mathcal{M}} N_m = |\mathcal{N}|$. These FBSs adopt a closed access control mechanism which only allows registered FUEs to use the FBSs' services [15]. For notational convenience, the MBS is indexed by 0. The OFDMA system has a bandwidth of B divided into a set $\mathcal{K} = \{1, 2, ..., K\}$ of subchannels which are reused at the CFN using underlay spectrum access model. These subchannels are correspondingly occupied by K macrocell user equipments (MUEs). All subchannels are assumed to be independent block fading channels and orthogonal subchannels. The channel fading is assumed to follow an i.i.d Rayleigh channel model. FBSs are connected to a cognitive femtocell management (CFM) that acts a coordinator and spectrum manager. We let $\mathcal{K}_m \subset \mathcal{K}$ be the set of subchannel available for FBS m as allocated by the CFM. FBSs and FUEs are assumed to be selfish and rational entities that seedk to maximize their individual objectives. Moreover, we assume that the FBSs and the MBS have knowledge about channel state information of FUEs. For convenience, a summary of the notations used is shown in Table I.

Table I: Summary of the notations.

Notation	Meaning
$\mathcal{M}$	Set of FBSs.
$\mathcal{N}$	Set of FUEs.
$\mathcal{N}_m$	Set of FUEs belonging to FBS m.
$\mathcal{K}$	Set of subchannels or MUEs.
$\mathcal{K}_m$	Set of subchannel available for FBS m.
$\mathcal{G}_k$	Set of FUE-FBS pairs allocating subchannel k.
μ_m	Matching game for subchannel allocation in FBS m.
$\succ_{n,m}$	Preference relation of FUE n in FBS m.
$\succ_{k,m}$	Preference list of subchannel k.
$\boldsymbol{Y}$	Matching matrix with $\{y^k_{n,m}\}$ elements.
$\boldsymbol{P}$	Transmit power matrix with $\{P^k_{n,m}\}$ elements.

B. Problem formulation

We first consider the FUE' QoS demand and the MBS protection. Then, we formulate the problem of optimal subchannel assignment and power allocation.

FUE QoS. We consider the minimum data rate requirement of the served FUEs. When FUE n is served by FBS m on a subchannel k with transmit power $P^k_{n,m}$, the data rate of FUE n will be given by

$$R^k_{n,m}(\boldsymbol{Y}, \boldsymbol{P}) = B_k \log_2(1 + \Gamma^k_{n,m}(\boldsymbol{Y}, \boldsymbol{P})), \quad (1)$$

in which B_k is bandwidth of subchannel k, $\Gamma^k_{n,m}(\boldsymbol{Y}, \boldsymbol{P})$ is the signal-to-interference-plus-noise ratio (SINR) of FUE n associated with FBS m on subchannel k.

The SINR $\Gamma^k_{n,m}(\boldsymbol{Y}, \boldsymbol{P})$ is given by

$$\Gamma^k_{n,m}(\boldsymbol{Y}, \boldsymbol{P}) = \frac{y^k_{m,n} g^k_{n,m} P^k_{n,m}}{I^{k,\text{in}}_{n,m} + g^k_{k,m} P^k_{k,0} + \sigma^2}, \quad (2)$$

where $I^{k,\text{in}}_{n,m} = \sum_{m'=1, m'\neq m}^{M} \sum_{n'=1}^{N_{m'}} y^k_{m',n'} g^k_{n'm} P^k_{n',m'}$ is the total interference from other femtocells to FBS m on subchannel k; $P^k_{n,m}$ and $P^k_{n',m'}$ are transmit powers of FUE $n \in \mathcal{N}_m$ and FUE $n' \in \mathcal{N}_{m'}, (m' \neq m)$ on subchannel k, respectively; $P^k_{k,0}$ is transmit power of MUE k on subchannel k; $g^k_{k,m}$ is the channel power gain on subchannel k from MUE k to FBS m; $g^k_{n,m}$ and $g^k_{n'm}$ are, respectively, channel power gains on subchannel k from FUE $n \in \mathcal{N}_m$ and from FUE $n' \in \mathcal{N}_{m'}, (m' \neq m)$ to FBS m; $\boldsymbol{Y} = [y^k_{m,n}]_{M\times N\times K}$ be the subchannel allocation matrix, where $y^k_{m,n} = 1$ means that subchannel k is assigned to FUE n, and $y^k_{m,n} = 0$ otherwise. Moreover, $\boldsymbol{P} = [P^k_{m,n}]_{M\times N\times K}$ is the power allocation matrix.

Without loss of generality, the noise power σ^2 are assumed to be equal at all FBSs.

To satisfy the QoS of FUEs, we assume that capacity of each FUE must be greater than a minimum rate requirement that is defined as follows:

$$R^k_{n,m}(\boldsymbol{Y}, \boldsymbol{P}) \geq R^{\text{min}}_n, \tag{3}$$

where R^{min}_n is predefined parameter of the FUE n.

MBS protection. In our model, each MUE seeks to transmit its data with fixed power level under a given QoS constraint such as delay or SINR. In addition, the MUE's QoS demand is predefined by converting to the total interference from FUEs to the MBS on each subchannel k with threshold $I^{k,\text{th}}$. In order to protect the MBS on subchannel k, the following condition must be satisfied:

$$\sum_{m=1}^{M}\sum_{n=1}^{N_m} y^k_{n,m} g^k_{n,0} P^k_{n,m} \leq I^{k,\text{th}}_0, \forall k \in \mathcal{K}, \tag{4}$$

where $\sum\limits_{m=1}^{M}\sum\limits_{n=1}^{N_m} y^k_{n,m} g^k_{n,0} P^k_{n,m}$ is the total interference generated by all FUEs to the MBS on subchannel k and $g^k_{n,0}$ is the channel power gain on subchannel k from FUE n to MBS.

Next, the problem of subchannel allocation and power control is formulated as an optimization problem which seeks to maximize the overall network throughput, as follows:

OPT:

$$\max_{(\boldsymbol{Y},\boldsymbol{P})} \sum_{m=1}^{M}\sum_{n=1}^{N_m}\sum_{k=1}^{K} R^k_{n,m}(\boldsymbol{Y}, \boldsymbol{P}) \tag{5}$$

$$\text{s.t.} \quad (3), (4),$$

$$\sum_{n=1}^{N_m} y^k_{n,m} \leq 1, \quad \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, \tag{6}$$

$$\sum_{k=1}^{K} y^k_{n,m} \leq 1, \quad \forall n \in \mathcal{N}_m, \forall m \in \mathcal{M} \tag{7}$$

$$P^{\text{min}}_n \leq P^k_{n,m} \leq P^{\text{max}}_n, \quad \forall n, m, k, \tag{8}$$

$$y^k_{n,m} = \{0, 1\}, \forall m, n, k. \tag{9}$$

Here, constraints (6) and (7) imply that each subchannel can be allocated to multiple FUEs that belongs to different FBSs and each FUE can be allocated at most one subchannel, respectively; the constraint (8) guarantees that the transmit power of each FUE on each subchannel is restricted by the FUE's transmit power limitation.

III. Resource Allocation as a Matching Game

The problem in (5) can be shown to be an NP-hard combinatorial optimization problem, because it contains binary variables ($\boldsymbol{Y}$) and continuous variables ($\boldsymbol{Y}$). To solve (5), we first decouple it by splitting into two independent phases: subchannel allocation phase (SCA) (between FUEs and subchannels), and power control phase (PC), as shown in Fig. 2. In order to solve problems in the SCA phase, we use an approach based on matching theory [16]. In the PC phase, we find the optimal

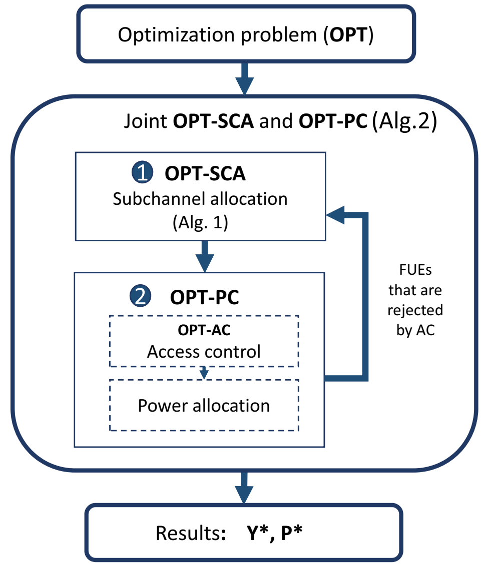


Figure 2: Proposed framework for solving the optimization problem in (5).

transmit power by using geometric programming. In addition, we consider an access control scheme to guarantee feasible solution in the PC phase. Then, we propose a framework to jointly combine SCA and PC phases.

A. Sub-channel allocation as a matching problem

In this subsection, we decouple variable $P^k_{n,m}$ and $y^k_{n,m}$ by fixing variable $\boldsymbol{Y}$, then the subchannels allocation $\boldsymbol{Y}$ are determined by solving optimization problem OPT-SCA, as follows:

OPT-SCA:

$$\max_{(\boldsymbol{Y})} \sum_{m=1}^{M}\sum_{n=1}^{N_m}\sum_{k=1}^{K} R^k_{n,m}(\boldsymbol{Y}, \boldsymbol{P}), \tag{10}$$

$$\text{s.t.} \quad (6), (7).$$

In problem OPT-SCA, (4) and (3), which will be considered in the PC phase, are temporarily ignored. Obviously, OPT-SCA is still NP hard. In order to solve it, we decompose OPT-SCA into M subproblems such that each subproblem corresponds to the subchannel allocation in signal FBS given allocated subchannels in other SBSs as follows:

OPT-SCA$_{\{m\}}$:

$$\max_{(\boldsymbol{Y})} \sum_{n=1}^{N_m}\sum_{k=1}^{K} R^k_{n,m}(\boldsymbol{Y}, \boldsymbol{P}), \tag{11}$$

$$\text{s.t.} \quad (6), (7).$$

Here, the problem OPT-SCA$_{\{m\}}$ is a combinatorial optimization problem with binary variables $y^k_{n,m}$, which can be solved centrally using the Hungarian algorithm. However, in our model, FUEs and FBSs' decision selfishly and rationally interact in a way that maximizing their utilities, which are also depend on resource allocation in the whole system. Therefore, in order to model competition among FUEs and FBSs, we solve the problem OPT-SCA$_{\{m\}}$ using a one-to-one matching game [17], [18], which helps us to find subchannel allocation in a distributed manner.

1) Definition of matching game for subchannel allocation of signal FBS in the SCA phase: The problem in (11) is formulated as a matching game which is defined by a tuple $(\mathcal{N}_m, \mathcal{K}_m, \succ_{\mathcal{N}_m}, \succ_{\mathcal{K}_m})$. Here, $\succ_{\mathcal{N}_m} = \{\succ_{n,m}\}_{n\in\mathcal{N}_m}$ and $\succ_{\mathcal{K}_m} = \{\succ_{k,m}\}_{k\in\mathcal{K}_m}$ denote the preference relation of FUEs and subchannels in FBS m, respectively. We define the problem as matching game one-to-one, as follows:

Definition 1. *Given two disjoint finite sets of FUEs $\mathcal{N}_m$ and subchannels $\mathcal{K}_m$, a matching game for subchannel allocation in FBS m is defined as a function μ_m: $\mathcal{N}_m \mapsto \mathcal{K}_m$ such that:*
1, $n = \mu_m(k) \leftrightarrow k = \mu_m(n)$, $n \in \mathcal{N}_m, k \in \mathcal{K}_m$;
2, $|\mu_m(k)| \leq 1$ and $|\mu_m(n)| \leq 1$, $n \in \mathcal{N}_m, k \in \mathcal{K}_m$.

The conditions $|\mu_m(k)| \leq 1$ and $|\mu_m(n)| \leq 1$ in Definition 1 correspond to the constraints in (6) and (7), respectively. In the matching μ_m, we define $\phi_{n,m}(k)$ and $\phi_{k,m}(n)$ are utility functions of FUE n for subchannel k and subchannel k for FUE n in FBS m, respectively. FUE n that associated with FBS m prefers subchannel k_1 to k_2 and an subchannel k in FBS m prefers FUE n_1 to n_2 are denoted by $k_1 \succ_{n,m} k_2$ $(k_1, k_2 \in \mathcal{K}_m)$ and $n_1 \succ_{k,m} n_2$ $(n_1, n_2 \in \mathcal{N}_m)$, respectively. Next, we define the utility function of both the FUE and FBS as bellows.

Utility function of the FUE. After the FUE associates with a FBS, each FUE obtains the corresponding subchannel response of each subchannel via a utility function which is proposed as follows:

$$\phi_{n,m}(k) = R^k_{n,m}(\boldsymbol{Y}, \boldsymbol{P}), \tag{12}$$

in which each FUE n estimates its utility on each subchannel k based on its data rate achieved on subchannel k. By using utility function in (12), FUEs have to bid to occupy each subchannel which maximize their utility function.

Utility function of the FBS on each subchannel. Each FBS tries to maximize its utility function on each subchannel which is given by

$$\phi_{k,m}(n) = \varphi_{\text{SCA}} R^k_{n,m}(\boldsymbol{Y}, \boldsymbol{P}) - C^k_{n,m}, \tag{13}$$

in which $R^k_{n,m}(\boldsymbol{Y}, \boldsymbol{P})$ is the estimation of the data rate of FUE n on subchannel k; φ_{SCA} is factor to convert data rate $R^k_{n,m}$ to benefit received from FUE n on subchannel k; $C^k_{n,m}$ is the cost due to interference caused by FUE n to the MBS and other FBSs on subchannel k, which is determined as follows:

$$C^k_{n,m} = c^k_0 \eta^k g^k_{n,0} P^k_{n,m} + \sum_{m'=1}^{M\setminus\{m\}} \sum_{n'=1}^{N_{m'}} y^k_{m',n'} c^k_{m'} g^k_{n,m'} P^k_{n,m}, \tag{14}$$

where $c^k_0 \eta^k g^k_{n0} P^k_{n,m}$ is the cost of violation at the MBS on subchannel k causing by FUE n given transmit power $P^k_{n,m}$; $\eta^k = \max(0, I^{k,\text{th}} - \sum_{m=1}^{M} \sum_{n=1}^{N_m} y^k_{n,m} g^k_{n,0} P^k_{n,m})$ is the metric which quantifies the degree of violation at the MBS on subchannel k which is proposed in [9]; $\sum_{m'=1, m'\neq m}^{M} \sum_{n'=1}^{N_{m'}} y^k_{m',n'} c^k_{m'} g^k_{n,m'} P^k_{n,m}$ quantifies the aggregate relative interference that FUE n causes to other FBS m' $(\forall m' \neq m)$ on subchannel k; c^k_0 and $c^k_{m'}$ are the cost per each interference transmit power unit at the MBS and FBS m', respectively. In our proposed matching game, each FBS m prefers to assign its subchannel to an FUE that maximizes FUE's satisfaction but less violation to the macrocell network and aggregate interference to other FBS on each subchannel.

Algorithm 1 (MSCA): Matching game algorithm for allocating subchannels.

Initialization:
1: $\mathcal{N}_m, \mathcal{K}_m, \mathcal{N}_k^{\text{req}} = \emptyset$, $\mathcal{N}_k^{\text{rej}} = \emptyset$, $\forall k \in \mathcal{K}_m$, $m =\in \mathcal{M}$.
2: Construct preference relation of all FUEs on subchannels that are not matched in each FBS using (12).
Swap matching to find stable matching μ_m:
3: **while** $\sum_{\forall k,n} b^{\text{SCA}}_{n\to k}(t) \neq 0$ **do**
4: *Each FUE $n \in \mathcal{N}_m$*:
5: Finds $k = \arg\max_{k\in\succ_{n,m}} \phi_{n,m}(k), \forall k \in \mathcal{K}_m$.
6: Sends a bid $b^{\text{SCA}}_{n\to k}(t) = 1$ to FBS m.
7: *Each subchannel k of FBS m ($k \in \mathcal{K}_m$)*:
8: Update bidder list on each subchannel k $\mathcal{N}_m^{k,\text{req}} \leftarrow \{n : b^{\text{SCA}}_{n\to k}(t) = 1, n \in \mathcal{N}_m\}$.
9: Construct preference list $\succ_{k,m}$ based on (13) .
10: Assign subchannel k to FUE n^* = $\arg\max_{n\in\succ_{k,m}} \phi_{k,m}(n)$.
11: Update reject list: $\mathcal{N}_m^{k,\text{rej}} \leftarrow \mathcal{N}_m^{k,\text{rej}} \cup \{\mathcal{N}_m^{k,\text{req}} \setminus \{n^*\}\}$.
12: Remove subchannel k from $\succ_{n,m}$, $\forall n \in \mathcal{N}_m^{k,\text{rej}}$.
13: **end while**
until: Convergence to stable matching μ^*_m.

2) Distributed algorithm for subchannel allocation based on matching game: For formulated one-to-one matching game, our purpose is to find a stable matching which is defined as follows:

Definition 2. *A matching μ^*_m is* stable *if there is no blocking pair. A pair $(n, k) \neq \mu_m$, where $n \in \mathcal{N}_m, k \in \mathcal{K}_m$ is said to be a blocking pair for the matching μ_m if there exists another matching $\mu'_m \in \mu_m(n, k)$ such that FUE n and FBS m can achieve a higher utility. Here, $\mu'_m \succ_{n,m} \mu_m$ and $\mu'_m \succ_k \mu_m$.*

The distributed algorithm used to solve the OPT-SCA$_{\{m\}}$ is presented in Algorithm 1 and referred to as the MSCA algorithm. In this algorithm, FBS m broadcasts its available subchannel to its FUEs. Based on information of subchannel k, each FUE constructs its preference list based on (12) (line 2). In the swap matching phase, each FUE sends a bid request $b^{\text{SCA}}_{n\to k}(t) = 1$ to access subchannel k that has the highest utility value (lines 3, 4 and 5). At the FBS side, the FBS collects information from bidding requests and constructs preference list on each subchannel (lines 8, 9 and 10). Based on the preference relation of subchannels, the FBS assigns subchannels to FUEs which bring highest utility value (line 11). The FUE removes the subchannel in its preference that is rejected by FBS m (line 12). In a signal FBS m, the process of acceptance or rejection of applicants is done in a manner analogous to the conventional deferred acceptance algorithms [17], [18]. Thus, Algorithm 1 in signal FBS m can converge to the stable matching μ_m^* [19].

B. Access control and power allocation in the PC phase

Since allocating subchannels to FUEs in a FBS can overlap with other, some FUEs in different FBSs can be allocated the same subchannel. To mitigate the cross interference among the FBSs and improve efficient spectrum usages, subchannel and power allocation among femtocells need to be coordinated. In order to coordinate among femtocells, FBSs send their proposals of subchannels to the CFM. The CFM collects information from FBS and then make decisions of subchannel and power allocation to proposed femtocells.

Due to orthogonal subchannels, we decompose the coordinated problems into K sub-problems. Each subproblem is given as follows:

OPT-PC$_{\{k\},k\in\mathcal{K}}$:
$$\max_{\boldsymbol{P}_{(\mathcal{G}_k)}} \sum_{n\in\mathcal{G}_k} R_{n,m}^k(\boldsymbol{P}_{(\mathcal{G}_k)}) \quad \text{s.t.} \quad (3),(4),(8). \tag{15}$$

Here, we only consider the FUEs and FBSs that are using the same subchannel k which is denoted by the set $\mathcal{G}_k$. OPT-PC$_{\{k\},k\in\mathcal{K}}$ is commonly known as the problem of joint power and admission control of the FUEs based on spectrum underlay, which try to find and admit a subset of FUEs to optimize different objectives [4], [5], [9]. These objectives include maximizing the number of admitted FUEs in set $\mathcal{G}_k$ and maximizing the total throughput in (15). The problem of maximizing number of active FUEs on the subchannel k is formulated as follows:

OPT-AC$_{\{k\},k\in\mathcal{K}}$:
$$\max.\ |\mathcal{G}_k| \quad \text{s.t.}\ (3),(4),(8). \tag{16}$$

Since OPT-AC$_{\{k\},k\in\mathcal{K}}$ is well-studied in the literature with many existing schemes for finding the optimal solutions [4], [5], [9], we adopt an algorithm known effective link gain radio removal algorithms (ELGRA) which is proposed in [5]. ELGRA is proved to obtain the globally optimal of the minimum outage problem stated in the OPT-AC$_{\{k\},k\in\mathcal{K}}$ with a computational complexity $O(|\mathcal{G}_k|\log|\mathcal{G}_k|)$.

Once a maximal feasible subset $\mathcal{G}_k$ is found, what remains is to adapt the transmit-power $\boldsymbol{P}_{(\mathcal{G}_k)}$ of the admitted FUEs so that the OPT-PC$_{\{k\},k\in\mathcal{K}}$ problem is maximized. Different with previous works, we solve OPT-PC$_{\{k\},k\in\mathcal{K}}$ using geometric programming [20].

Previously, we transform the OPT-PC$_{\{k\},k\in\mathcal{K}}$ into convex problem. Since $R_{n,m}^k >> 0$, $\Gamma_{n,m}^k >> 1$, we have $R_{n,m}^k \approx B_k\log(\Gamma_{n,m}^k)$. Therefore, $\sum_{n\in\mathcal{G}_k} R_{n,m}^k \approx \sum_{n\in\mathcal{G}_k} B_k\log_2\left(\Gamma_{n,m}^k\right)$. Then, the OPT-PC is equivalent to the following problem:

OPT-PC$_{\{k\},k\in\mathcal{K}}$:
$$\min.\ \sum_{n\in\mathcal{G}_k} B_k\log_2\left(\frac{1}{\Gamma_{n,m}^k}\right), \quad \text{s.t.}\ (3),(4),(8). \tag{17}$$

This optimization problem is now transformed into a standard form of the geometric programming problem, which remains non-convex. However, by defining a new variable $\tilde{P}_{n,m}^k = \log P_{n,m}^k$ and a new feasible set $\mathcal{V}_n^k = \{P_{n,m}^k | P_{n,m}^k \in [\log P_n^{\min},\ \log P_n^{\max}],\ \forall n \in \mathcal{G}_k\}$. Additionally, we introduce an auxiliary variable to estimate intra-tier interference $Z_{n',m}^k \triangleq g_{n',m}^k P_{n',m'}^k$ and an new variable $\tilde{Z}_{n',m}^k = \log(Z_{n',m}^k)$, where $n\in\mathcal{N}_m, n'\in\mathcal{N}_{m'}$, $n,n',m,m'\in\mathcal{G}_k, m'\neq m$. The OPT-PC$_{\{k\},k\in\mathcal{K}}$ becomes:

OPT-PC-1$_{\{k\},k\in\mathcal{K}}$:
$$\min_{(\tilde{\boldsymbol{P}},\tilde{\boldsymbol{Z}})} \sum_{n=1}^{|\mathcal{G}_k|}\log\left[\frac{e^{-\tilde{P}_{n,m}^k}}{g_{n,m}^k}\left(\sum_{n'=1}^{|\mathcal{G}_k\setminus\{n\}|} e^{\tilde{Z}_{n',m}^k} + g_{k,m}^k P_{k,0}^k + \sigma^2\right)\right], \tag{18}$$

s.t.
$$\sum_{n=1}^{|\mathcal{G}_k|} g_{n,0}^k e^{\tilde{P}_{n,m}^k} - I_0^{k,\text{th}} \leq 0, \forall n,m\in\mathcal{G}_k, n\in\mathcal{N}_m, \tag{19}$$
$$\log\left[\frac{e^{-\tilde{P}_{n,m}^k}}{g_{n,m}^k}\left(\sum_{n'=1,n'\neq n}^{|\mathcal{G}_k|} e^{\tilde{Z}_{n',m}^k} + g_{k,m}^k P_{k,0}^k + \sigma^2\right)\right] - \log(\chi_n) \leq 0, n,n'\in\mathcal{G}_k, n\in\mathcal{N}_m, n'\in\mathcal{N}_{m'}, m'\neq m. \tag{20}$$
$$\tilde{Z}_{n',m}^k = \log(g_{n',m}^k) + \tilde{P}_{n',m'}^k, \forall n,m\in\mathcal{G}_k, m'\neq m, \tag{21}$$
$$\tilde{P}_{n,m}^k \in \mathcal{V}_n^k, \forall n,m\in\mathcal{G}_k, k\in\mathcal{K}, \tag{22}$$

in which $\chi_n = 2^{-\frac{R_n^{\min}}{B_k}}$, $\forall m,m'\in\mathcal{M}$.

We can see that OPT-PC-1$_{\{k\},k\in\mathcal{K}}$ is a convex optimization problem in the $(\tilde{\boldsymbol{P}},\tilde{\mathbf{Z}})$-space [21]. The optimization problem OPT-PC-1$_{\{k\},k\in\mathcal{K}}$ is now a standard optimization problem that can be solved using solvers such as the YALMIP toolbox to find optimal transmit power of FUEs [22].

We note that the optimization problems OPT-AC$_{\{k\},k\in\mathcal{K}}$ and OPT-PC-1$_{\{k\},k\in\mathcal{K}}$ can be solved by the CFM. The CFM plays a role as an access controller to make decisions in rejecting or accepting FUEs that are assigned subchannels in the SCA phase. Additionally, the CFM also plays a role as an optimizer for OPT-PC-1$_{\{k\},k\in\mathcal{K}}$.

C. Joint subchannel allocation and power control

In this subsection, we propose a framework for joint SCA and PC as shown in Fig. 2. The proposed framework is discussed in Algorithm 2. In the initialization step, each FBS m contains a set of $\mathcal{K}$ available subchannels that are not matched to its FUEs. Then, $P_{n,m}^k$ is initialized with uniform power distributed among all available subchannels. First of all, FUEs join into the SCA phase to find subchannel allocation based on Algorithm 1. Then, FBSs send proposals of their subchannels

Algorithm 2 : Joint SCA and PC.

Initilization:

1: Initialize $P^k_{n,m}$ with uniform power distributed among all available subchannels, $\forall n \in \mathcal{N}_m$, $\mathcal{K}_m = \mathcal{K}$, $\forall m \in \mathcal{M}$.
2: **repeat**
3: *Each* FBS $m(m \in \mathcal{M})$:
4: Runs the Algorithm 1 to find subchannel allocation.
5: Sends FUE's proposal on subchannel k to the AC.
6: *At the access controller (on each subchannel $k(k \in \mathcal{K})$):*
7: Collect information FUEs allocating to subchannel $k, (k \in \mathcal{K})$.
8: **while** OPT-PC$_{\{k\},k\in\mathcal{K}}$ is infeasible solution **do**
9: Removes pairs $(n, m) \in \mathcal{G}_k$ based on ELGRA algorithm.
10: Removes subchannel k from $\succ_{n,m}$ in FBS m.
11: **end while**.
12: Update subchannel allocations information.
13: Update transmit power for FUEs using YALMIP tool.
14: Goes back to line 4 .
15: **until** Convergence to stable groups $\mathcal{G}_k, \forall k \in \mathcal{K}$.

in the SCA phase to the CFM. The CFM rejects (FUE-FBS) pairs proposed on subchannel k that cause infeasible solution in OPT-PC-1$_{\{k\},k\in\mathcal{K}}$ based on the ELGRA algorithm (line 9). The FUEs that are removed by the access controller will find new subchannels in their preference list by going back to the MSCA algorithm in the SCA phase (line 14). On the other hand, FUEs that guaranteed feasible solution of problem OPT-PC will be allocated transmit power by using YALMIP toolbox (line 13). Next, subchannel and power allocation are updated for all FUEs at all femtocells. Due to new updated information leads to changing preference relation of FUEs and FBSs, the SCA will be repeated to find subchannel allocation for FUEs that not allocated any subchannel. Our algorithm repeats until $\mathcal{G}_k, \forall k \in \mathcal{K}$ remain unchanged for two consecutive matchings or have no new request from FUEs in both UA and SCA phases which mean achieve a group state. The convergence and stability of the Algorithm 2 are proved in Section IV.

IV. Convergence and stability of the Proposed algorithm

In this section, we prove that Algorithm 2 can converge to group stable matching $\mathcal{G}_k, \forall k \in \mathcal{K}$. We define the group stability and show that our proposed algorithms results in group stable matchings as below.

Definition 3. *Given the interrelationship between FUEs, FBS, and subchannels in the Algorithm 2, a group $\mathcal{G}_k$ is stable if it is not blocked by any group as expressed via following two conditions:*
1) No FUE n' outside the group $\mathcal{G}_k$ can join it.
2) No FUE n inside the group $\mathcal{G}_k$ can leave it.

A matching is group stable *if and only if all the groups $\mathcal{G}_k, \forall k \in \mathcal{K}$ are group stable.*

We consider a group of (n, m) pairs $\mathcal{G}_k$ that is formed by matching the FUEs to subchannels in FBSs and optimization processes in the PC phase. Assuming there exist a FUE $n' \in \mathcal{N}\backslash\mathcal{G}_k$. We first consider $n', n \in \mathcal{N}_m$. Due to n' can join into group $\mathcal{G}_k$, $n' = \mu_m(k)$. Due to matching μ_m is stable, $n' \equiv n$ and $n = \mu_m(k)$. Next, we consider the scenario $n' \in \mathcal{N}_{m'}, n \in \mathcal{N}_m, m \neq m'$. If n' can join to group $\mathcal{G}_k$, $n' = \mu_m(k)$ in stable matching $\mu_m(k)$ of FBS m'. Additionally, given proposal of n' in FBS m to the PC phase, n' can join to $\mathcal{G}_k$ if $\sum_{(n',m')\rightarrow\mathcal{G}^k} R^k_{n,m} > \sum_{\mathcal{G}^k} R^k_{n,m}$. But due to $\mathcal{G}_k$ is formed by stable matchings in the SCA phase, no more FUEs are requested in the SCA phase which lead to no more FUE are rejected in the PC phase. Hence, $(n', m') = \arg\max \sum_{(n,m)\in\mathcal{G}^k} R^k_{n,m}$. Therefore, from two above considered scenarios, we can say that $n' \equiv n$ or no FUE n' outside the group $\mathcal{G}_k$ can join it. Similarly, no FUE n inside the group $\mathcal{G}_k$ can leave it.

However, the two conditions in the Definition 3 are not sufficient to ensure the required stability of the matching.

Proposition 1. *Given the matching operation in the Algorithm 2, the FUE n that is accepted by matching μ_m in the SCA phase and processing optimization in the PC phase will not be rejected in plan of any new applicant in next iterations.*

Proof: First, in the initial iteration where no FUE are matched to any subchannel. When forming a group $\mathcal{G}_k$, each FUE in $\mathcal{G}_k$ is accepted in the SCA with FUE $n = \arg\max_{n\in\succ_k} \phi^k_{\text{SCA}}(n)$ and PC phase with FUE $n = \arg\max_{n\in\mathcal{G}^k} \sum_{(n,m)\in\mathcal{G}^k} R^k_{n,m}$. Thus, the preference ordering of FUE n in the SCA phase of each FBS m can be denoted by $\mathcal{U} \succ_n k \succ_n \mathcal{W}$ in which $\mathcal{U}$ and $\mathcal{W}$ are defined as follows:

$$\mathcal{U} = \{\forall k' \in \mathcal{K}_m | \phi^k_{\text{SCA}}(n') > \phi^k_{\text{SCA}}(n)\} \tag{23}$$
$$\mathcal{W} = \{\forall k' \in \mathcal{K}_m | \phi^k_{\text{SCA}}(n') < \phi^k_{\text{SCA}}(n)\} \tag{24}$$

In the next iterations when FUEs that are rejected by CFM in optimization processes of the PC phase, the processes of acceptance or rejection of new applicants in the SCA phase are restarted using MSCA algorithm. Thus, we can ensure each $k \in \mathcal{K}_m$ is accepted in previous iteration, the FUE that $\mu_m(k) = \arg\max_{n\in\succ_k} \phi^k_{\text{SCA}}(n)$. So, if FUE $n' \neq n$ applies for subchannel k in the next matching μ'_m of next iterations, then $k' = \mu'_m(n') \in \mathcal{U}$. Now, we can conclude that the FUEs who are accepted in the first iteration will not be rejected by their match as the game proceeds. ■

From the Proposition 1, we show the stability of matchings in the Algorithm 2 as follows:

Theorem 1. *Each group $\mathcal{G}_k, k \in \mathcal{K}$ becomes a group stable after finite number of iterations and, thus the Algorithm 2 is guaranteed to converge.*

Proof: Because the detected FBS by each FUE and available subchannel in each FBSs are finite, preference lists of each FUE in the SCA phase are also finite. Moreover, each FUE $n \in \mathcal{N}_m, \forall m \in \mathcal{M}$ will not reject its current match $\mu_m(k)$ which discussed in Proposition 1. Hence, only new FUEs can join a subchannel k that are not matched with any other FUEs. Thus, FUE n will not get back to a subchannel if it is rejected in previous iterations. Additionally, have no new proposal sent by FUE n in FBS m on subchannel k when

FUE n is rejected in the PC phase in previous iterations. Therefore, the each rejecting from the FBS and CFM side cause decreasing size of its preference lists in the SCA phase. Therefore, each group $\mathcal{G}_k$ are formed after a finite number of iterations. ■

Therefore, given Proposition 1 and Theorem 1, they are enough to show that any matched pair (n, k) will not achieve a higher utility than if any entity in the matched pair were to match in other pairs. Hence, the proposed Algorithm 2 is guaranteed to reach a stable outcome for matching FUEs and subchannels in all FBSs after a finite number of iterations.

V. SIMULATION RESULTS

For our simulations, we consider one MBS and 5 FBSs with the coverage radii of 500 m and 25 m, respectively. FBSs are deployed in a small indoor area 150 m ×125 m to serve 4 FUEs on each FBS [7]. In the CFN, we consider 5 orthogonal subchannels, which are allocated to 5 MUEs in the macrocell network. The bandwidth of each subchannel is 360 kHz and MUEs have fixed power level of 100 mW. The constraint of maximum interference power on each subchannel at the MBS is -70 dBm. The noise variance is set to -110 dBm. The slow-fading channel gain is assumed to be i.i.d Rayleigh distributed random variables with mean value $g(d) = g_0(d/25)^{-4})$ where g_0 is a reference channel gain at a distance 25 m. Each FUEs has minimum rate requirement of 2.5 Mbps. Each FUE has maximum transmit power of 100 mW. Moreover, we set the values φ_{SCA}, c_0^k, and c_m^k equal to 1, 1, and 0.1, respectively.

All statistical results are averaged over a large number of independent simulation runs. FUEs are randomly located inside the coverage region of FBSs. Moreover, MUEs are randomly distributed outside area 150×125 m^2.

In Section IV, we have shown that Algorithm 2 converges to a stable group if and only if the SCA phase is stable. Hence, without loss of generality, we consider the convergence and stability of our proposed algorithms via stable matching in the SCA phase. In Fig. 3a, the SCA phase converges to stable matching after around 15 iterations of the MSCA algorithm, which means that there exits no further requests from FUEs to FBSs after 15 iterations in the SCA phase. By changing the number of FUEs, we can see that the average number of requests is increasing with the number of FUEs in the system (shown in Fig. 3b). Moreover, Fig. 3b shows that the SCA phase converges to stable matching with small number of requests from FUEs.

To evaluate the performance, we compare our approach with three baselines. The first baseline is a “random” scheme in which FBSs and CFM randomly assign subchannels to FUEs. The second baseline is a “greedy” in which FUEs choose subchannels with highest data rate in SCA phase. The third baseline, is the PC phase without OPA scheme in which the subchannels are allocated to FUEs by using MSCA and ELGRA but we ignore finding optimal power allocation in the PC phase.

In Fig. 4, we show the average aggregate throughput of the uplink CFN versus the total number of FUEs, while fixing interference threshold on all subchannels $I^{k,\text{th}}$ = -70 dBm.

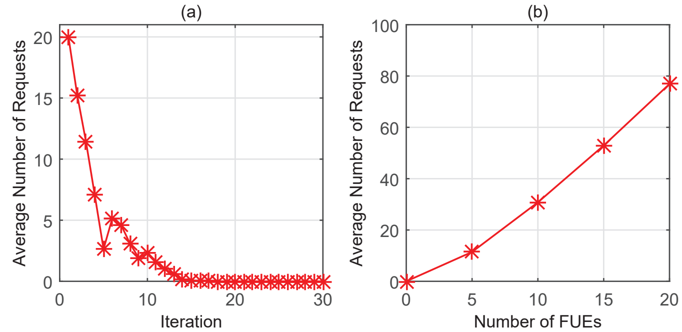


Figure 3: Convergence of the SCA phase in the Algorithm 2: (a) Convergence of the SCA phase with N = 20 FUEs; (b) The average total number of requests of the SCA phase versus the number of FUEs.

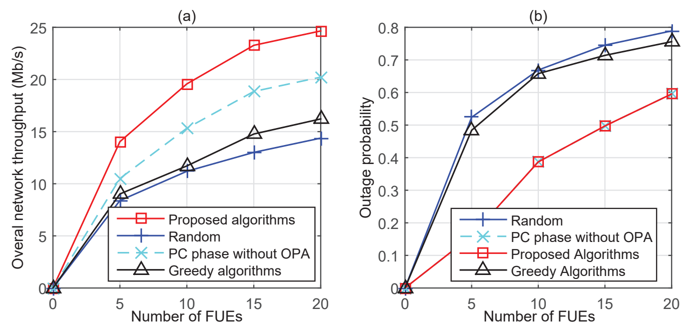
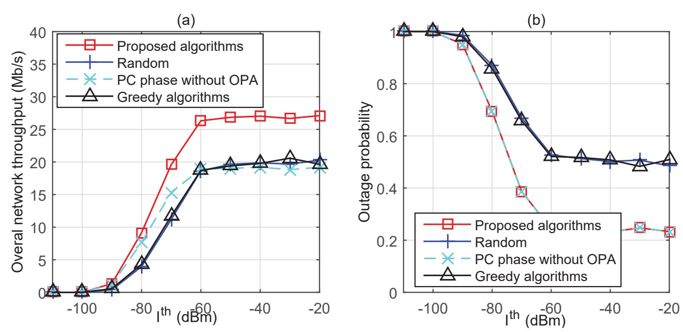


Figure 4: Total throughput and outage probability in uplink CFN versus number of FUEs N when M = 5, K = 5, $I^{k,\text{th}}$ = -70 dBm.



Fairness

Figure 5: Total throughput and outage probability in uplink CFN versus the interference threshold I_k^{th} when M = 5, N = 10 FUEs, K = 5.

As the N increases, the total throughput from the proposed algorithms and considered baselines increases. However, our proposal is better than other schemes in term of the total network throughput and outage probability. This is due to the fact that, as N increases, competition among FUEs to allocate subchannels increases. Using proposed algorithms, the FUEs will be more connected within groups which means more efficient subchannel allocation as shown in Fig. 4b. Moreover, based on the optimal power allocation of FUEs, proposed algorithms will be achieved maximum transmit power for FUEs while guaranteeing the MBS protection and minimum data rate of FUEs. Thus, our proposed scheme is also achieved more efficient power allocation than one another as showed in Fig. 4a.

Fig. 5a and Fig. 5b show the total uplink capacity and outage probability of the CFN for the interference temperature limit increases from -110 dBm to -20 dBm, respectively. It can be seen from the figure that higher interference threshold limit I_k^{th} induces higher total capacity and lower outage probability of the CFN. Fig. 5b shows that the total throughput and outage probability are up to around 40% higher for the proposed algorithms compare to the Greedy algorithm and random baselines for $I_0^{k,\mathrm{th}} = -20\mathrm{dBm}$, respectively. As the interference threshold $I_k^{k,\mathrm{th}}$ reduces, the transmit power of FUEs to avoid interference at MBS on subchannels reduces. Hence, the CFN resource become more scarce. However, Fig. 5b shows that by using the proposed algorithms, FUEs will be more connected which means more efficient subchannel allocation. Fig. 5a shows that by using the proposed algorithms, FUEs will be achieved higher total throughput under finding optimal power allocation than one another.

VI. Conclusions

In this paper, we have proposed a novel framework for joint subchannel assignment and power allocation problem in uplink cognitive femtocell network. Based on the studied model, the efficient resource allocation in the CFN has considered via an optimization problem in which we have maximized the total uplink throughput while guaranteeing FUEs minimum rate requirement and MBS protection. The optimization problem has shown to be NP-hard. In order to solve it, we have proposed a novel framework based on dynamic matching game. Then, we have designed distributed algorithms to find subchannel assignment and power allocation for FUEs. Our algorithms have proved converting to group stable matchings. Simulation results have showed that the proposed approach have yielded a performance improvement, in terms of the total network throughput and outage probability with a small number of iterations.

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